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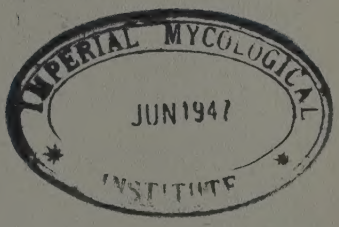

VOLUME XLV

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MATHÉMATIQUES

SUR LA VARIATION DE LA DILATATION D'UNE FONCTION RÉGULIÈRE

Par A. BERMANT

(Présenté par A. N. Kolmogoroff, de l'Académie, le 15.III.1944)

Considérons une fonction $w = f(z)$, régulière à l'intérieur du cercle $|z| < 1$ et vérifiant la condition $f(0) = 0$. Il est intéressant à certains points de vue d'étudier la variation de la dilatation d'une telle fonction au point $z = 0$, c'est-à-dire de la quantité $|f'(0)|$, quand on fait varier légèrement les valeurs limites de la fonction. On a dans cette direction le résultat suivant de M. Lavrentieff (1).

Soit D un domaine situé dans le plan de la variable complexe $w = \rho e^{i\theta}$, borné par une courbe simple fermée $\rho = \rho(\theta)$, $\rho(\theta + 2\pi) = \rho(\theta)$, jouissant des propriétés:

1) $|1 - \rho(\theta)| < k$, *où k est un nombre positif inférieur à $^1/_4$;*

2) *la fonction $\rho(\theta)$ vérifie la condition de Lipschitz $|\rho(\theta + \Delta\theta) - \rho(\theta)| < k_1 |\Delta\theta|$, où k_1 est une constante, $^1/_3 > k_1 > 2\pi k$.*

Dans ces conditions, si nous désignons par $z = F(w)$, $F(0) = 0$, la fonction réalisant la représentation conforme du domaine D sur le cercle $|z| < 1$, nous aurons

$$1 + \frac{1}{2\pi}(\sigma_2 - \sigma_1) - Kkk_1 |\ln k_1| < |F'(0)| < 1 + \frac{1}{2\pi}(\sigma_2 - \sigma_1) + Kkk_1 |\ln k_1| \qquad (1)$$

où σ_2 est l'aire du domaine que l'on obtient en prenant l'intersection du cercle $|w| < 1$ et de ce qui est extérieur à D, et σ_1 est l'aire de l'intersection des domaines $|w| > 1$ et D; K est une constante absolue.

Nous donnons ici une généralisation essentielle de ce résultat, et d'ailleurs dans la démonstration, tout à fait distincte de celle de M. Lavrentieff, nous nous appuyons sur une évaluation que nous avons trouvée(2) pour $|f'(0)|$ que nous exprimons au moyen de l'aire de l'étoile de la fonction $f(z)$ au point $w = f(0) = 0$. (Nous appelons étoile de la fonction $w = f(z)$ au point $w = f(0) = 0$ le plus grand domaine étoilé par rapport au point $w = f(0) = 0$ et appartenant à la surface définie par la fonction $f(z)$ quand z se trouve dans le cercle $|z| < 1$. L'évaluation indiquée $|f'(0)|$ est de la forme

$$|f'(0)| \leq \sqrt{S^*/\pi}$$

où S^* est l'aire de l'étoile).

Désignons par ε le plus grand des deux nombres k et k_1 et par $w = f(z)$ la fonction inverse $z = F(w)$. Alors, en tenant compte de ce que $\sigma_2 - \sigma_1$ est une quantité d'ordre ε, on peut écrire la formule (1) dans la forme

$$|f'(0)| = \sqrt{S/\pi} + o(\varepsilon^{2-\eta}), \quad \eta > 0,$$

où S est l'aire du domaine D.

Théorème. *Si la frontière λ_1^* de l'étoile de la fonction $w=\rho e^{i\theta}=f(z)$, $f(0)=0$, régulière dans le cercle $|z|<1$, $z=re^{i\varphi}$, appartient à l'anneau $1-\varepsilon\leqslant|w|\leqslant 1+\varepsilon$ et si l'on a pour λ_1^* l'inégalité $|\rho(\theta+\Delta\theta)-\rho(\theta)|\leqslant K\varepsilon|\Delta\theta|$ (sauf les segments radiaux qui appartiennent à λ_1^*), alors*

$$|f'(0)|=\sqrt{S^*/\pi}+(n+1)\,o(\varepsilon^{2-\eta}),\ \eta>0$$

où S^* est l'aire de l'étoile $f(z)$ au point $f(0)=0$, n est le nombre des segments radiaux qui appartiennent à λ_1^*.

Démonstration. Il est évident que pour évaluer $|f'(0)|$ nous pouvons supposer que la fonction $f(z)$ est régulière dans le cercle fermé $|z|\leqslant 1$; alors sur la frontière de l'étoile, sauf sur les n segments radiaux qui déterminent l'étoile, nous aurons $|\rho'(\theta)|\leqslant K\varepsilon$.

Désignons par $w=f^*(z)$ la fonction qui réalise la représentation conforme du cercle $|z|<1$ sur l'étoile de $f(z)$ et d'ailleurs telle que $f^*(0)=0$.

Soit

$$f^*(z)=a_1z+a_2z^2+\ldots+a_nz^n+\ldots$$

D'ailleurs, il est connu que

$$S^*=\frac{1}{2}\int_0^{2\pi}\rho^2d\theta=\pi\left[|a_1|^2+2|a_2|^2+\ldots+n|a_n|^2+\ldots\right]$$

et

$$S_1^*=\frac{1}{2}\int_0^{2\pi}\rho^2d\varphi=\pi\left[|a_1|^2+|a_2|^2+\ldots+|a_n|^2+\ldots\right]$$

Multiplions cette dernière égalité par le nombre 2 et soustrayons-la de l'égalité précédente; on a

$$S^*-2S_1^*=\pi\left[-|a_1|^2+|a_3|^2+\ldots+(n-2)|a_n|^2+\ldots\right]$$

Il en résulte que

$$|a_1|^2\geqslant\frac{1}{\pi}(2S_1^*-S^*)=\frac{1}{\pi}S^*+2\frac{1}{\pi}(S_1^*-S^*)$$

Or, en vertu du principe de Lindelöf,

$$|a_1|=|f^{*\prime}(0)|\leqslant|f'(0)|$$

et, par conséquent

$$|f'(0)|^2\geqslant\frac{1}{\pi}S^*+2\frac{1}{\pi}(S_1^*-S^*)$$

D'autre part, ainsi qu'il a été indiqué précédemment, nous avons

$$|f'(0)|^2\leqslant\frac{1}{\pi}S^*$$

Par conséquent

$$\frac{1}{\pi}S^*+2\frac{1}{\pi}(S_1^*-S^*)\leqslant|f'(0)|^2\leqslant\frac{1}{\pi}S^*$$

Evaluons le terme complémentaire $\frac{2}{\pi}(S_1^*-S^*)$. Nous avons

$$\left|\frac{2}{\pi}(S_1^*-S^*)\right|=\frac{1}{\pi}\left|\int_0^{2\pi}\rho^2d(\theta-\varphi)\right|=\frac{2}{\pi}\left|\int_0^{2\pi}(\theta-\varphi)\,\rho\rho'd\theta\right|$$

Comme on a sur la frontière de l'étoile $1-\varepsilon \leqslant \rho(\theta) \leqslant 1+\varepsilon$, il en résulte $|\theta-\varphi| \leqslant K_1 \varepsilon |\ln \varepsilon|$, où K_1 est une constante, ce qui suit des résultats de A. Martchenko[3].

D'ailleurs, $|\rho'(\theta)| \leqslant K\varepsilon$, et sur les segments radiaux on a $|d\rho| \leqslant 2\varepsilon$.

Par conséquent

$$\left|\frac{2}{\pi}(S_1^2 - S^*)\right| \leqslant K_2 (n+1)\varepsilon^2 |\ln \varepsilon|$$

où K_2 est une constante.

Nous obtenons

$$|f'(0)|^2 = \frac{1}{\pi} S^* + (n+1)\, o(\varepsilon^{2-\eta})$$

d'où finalement

$$|f'(0)| = \sqrt{S^*/\pi} + (n+1)\, o(\varepsilon^{2-\eta}) \qquad \eta > 0$$

ce qu'il fallait démontrer.

L'évaluation de la dilatation que nous venons de donner peut trouver plusieurs applications dans l'analyse qualitative des procédés de contractions des domaines et des surfaces riemanniennes vers le noyau; ces procédés ont été considérés au point de vue quantitatif par M. Carathéodory.

Institut Mathématique.
Académie des Sciences de l'URSS.

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MATHEMATICS

ON SOME ESTIMATIONS CONNECTED WITH SINGULAR INTEGRAL OF C. DE LA VALLÉE-POUSSIN

By I. P. NATANSON

(Communicated by V. I. Smirnov, Member of the Academy, 20.III.1944)

Let $f(t)$ be a continuous function periodic in 2π. Set

$$V_n(x)=\frac{1}{2\pi}\frac{(2n)!!}{(2n-1)!!}\int_{-\pi}^{\pi} f(t)\cos^{2n}\frac{t-x}{2}\,dt \tag{1}$$

This is the singular integral introduced by C. de la Vallée-Poussin. It is known that with n increasing, this integral tends uniformly to $f(x)$. The object of this note is the estimation of the difference $V_n(x)-f(x)$.

This problem is dealt with under some assumptions concerning the nature of the function $f(t)$.

Theorem 1. *If the second derivative $f''(x)$ exists and is finite at x, then*

$$\lim_{n\to\infty} n\,[V_n(x)-f(x)]=f''(x) \tag{2}$$

Proof. We have

$$f(t)=f(x)+f'(x)\sin(t-x)+\frac{f''(x)}{2}\sin^2(t-x)+\alpha(t)\sin^2(t-x) \tag{3}$$

where $\alpha(t)$ tends to zero with $t-x$.

Replacing $f(t)$ in (1) we obtain:

$$V_n(x)=f(x)+\frac{2n+1}{2n+2}\frac{f''(x)}{n+2}+\rho_n$$

where

$$\rho_n=\frac{1}{2\pi}\frac{(2n)!!}{(2n-1)!!}\int_{-\pi}^{\pi}\alpha(t)\sin^2(t-x)\cos^{2n}\frac{t-x}{2}\,dt.$$

It remains to show that

$$\lim_{n\to\infty} n\rho_n=0.$$

Taking into account that $\alpha(t)$ is periodic in 2π, we get

$$\rho_n=\frac{1}{\pi}\frac{(2n)!!}{(2n-1)!!}\int_{0}^{\pi/2}\beta(t)\sin^2 2t\cos^{2n}t\,dt \tag{4}$$

where $\beta(t)=\alpha(x+2t)+\alpha(x-2t)$. Now take $\varepsilon>0$ and choose $\delta>0$ so small that $|\beta(t)|<\varepsilon$ when $0<t<\delta$. Splitting up the integral (4) into two integrals extended over the intervals $[0, \delta]$ and $[\delta, \pi/2]$, we have

$$|\rho_n|<\frac{\varepsilon}{\pi}\frac{(2n)!!}{(2n-1)!!}\int_0^{\delta}\sin^2 2t\cos^{2n}t\,dt+\frac{\cos^{2n}\delta}{\pi}\frac{(2n)!!}{(2n-1)!!}\int_{\delta}^{\pi/2}|\beta(t)|\sin^2 2t\,dt$$

Putting $q=\cos^2\delta$, $k=\frac{1}{\pi}\int_0^{\pi/2}|\beta(t)|\sin^2 2t\,dt$, we obtain

$$|\rho_n|<\frac{\varepsilon}{n+2}+2knq^n$$

whence follows the theorem.

Theorem 2. *If the function $f(t)$ has $\omega(\delta)$ for its modulus of continuity, then*

$$|V_n(x)-f(x)|\leqslant 3\omega\left(1/\sqrt{n}\right)$$

Proof. It is evident that

$$|V_n(x)-f(x)|\leqslant\frac{1}{2\pi}\frac{(2n)!!}{(2n-1)!!}\int_{x-\pi}^{x+\pi}\omega(|t-x|)\cos^{2n}\frac{t-x}{2}\,dt$$

It is well known that the function $\omega(\delta)$ satisfies the condition $\omega(\lambda\delta)\leqslant$ $\leqslant(\lambda+1)\omega(\delta)$.

From this it follows that

$$|V_n(x)-f(x)|\leqslant\frac{1}{2\pi}\frac{(2n)!!}{(2n-1)!!}\omega\left(\frac{1}{\sqrt{n}}\right)\left\{\sqrt{n}\int_{x-\pi}^{x+\pi}|t-x|\cos^{2n}\frac{t-x}{2}\,dt+\right.$$

$$\left.+\int_{x-\pi}^{x+\pi}\cos^{2n}\frac{t-x}{2}\,dt\right\}$$

Hence

$$|V_n(x)-f(x)|\leqslant\frac{8n}{\pi}\omega\left(\frac{1}{\sqrt{n}}\right)\int_0^{\pi/2}t\cos^{2n}t\,dt+\omega\left(\frac{1}{\sqrt{n}}\right)$$

and

$$|V_n(x)-f(x)|\leqslant\left(\frac{4n}{2n+1}+1\right)\omega\left(\frac{1}{\sqrt{n}}\right)\leqslant 3\omega\left(\frac{1}{\sqrt{n}}\right)$$

The theorem is proved. In particular, from this theorem follows

Theorem 3. *If $f(t)$ satisfies the Lipschitz condition with exponent α, then*

$$|V_n(x)-f(x)|<k/n^{\alpha/2}$$

This proposition, however, can also be proved in a simpler way. In fact, assuming that

$$|f(t)-f(x)|\leqslant M|t-x|^{\alpha}$$

we obtain

$$|V_n(x)-f(x)|\leqslant\frac{M}{2\pi}\frac{(2n)!!}{(2n-1)!!}\int_{x-\pi}^{x+\pi}|t-x|^{\alpha}\cos^{2n}\frac{t-x}{2}\,dt$$

whence

$$|V_n(x)-f(x)|\leqslant\frac{M}{\pi^{1-\alpha}}\frac{(2n)!!}{(2n-1)!!}\int_0^{\pi/2}\sin^\alpha t\cos^{2n}t\,dt$$

We apply to this integral Hölder's inequality, to obtain

$$|V_n(x)-f(x)|\leqslant\frac{2M}{\pi^{1-\alpha}}\frac{(2n)!!}{(2n-1)!!}\left(\int_0^{\pi/2}\sin^2 t\cos^{2n}t\,dt\right)^{\alpha/2}\left(\int_0^{\pi/2}\cos^{2n}t\,dt\right)^{\frac{2-\alpha}{2}}$$

Consequently

$$|V_n(x)-f(x)|\leqslant\frac{M\pi^\alpha}{2^{\alpha/2}}\frac{1}{(n+1)^{\alpha/2}}<\frac{k}{n^{\alpha/2}}$$

As $\pi<3\sqrt{2}$, this argument leads to a better constant than Theorem 2.

Theorem 4. *There exists a function $f(t)$ satisfying the Lipschitz condition with the exponent α for which*

$$\max|V_n(x)-f(x)|>c/n^{\alpha/2}\qquad(c>0)$$

Such a function, for instance, is $f(t)=|\sin t|^\alpha$. With respect to this function we have

$$V_n(0)=\frac{1}{2\pi}\frac{(2n)!!}{(2n-1)!!}\int_{-\pi}^{\pi}|\sin t|^\alpha\cos^{2n}\frac{t}{2}\,dt$$

Hence

$$V_n(0)>c_1\sqrt{n}\int_0^{\pi/2}t^\alpha\cos^{2n}t\,dt>c_1\sqrt{n}\int_0^{1/\sqrt{n}}t^\alpha\cos^{2n}t\,pt$$

Consequently

$$V_n(0)>c_1\sqrt{n}\int_0^{1/\sqrt{n}}t^\alpha(1-nt^2)\,dt=c_2/n^{\alpha/2}$$

Thus, the theorem is proved.

A paper by M. Kac (') was published in the «Studia Mathematica» (volume 7) dealing with similar results in the theory of Bernstein's polynomials

$$B_n(x)=\sum_{k=0}^{n}f\left(\frac{k}{n}\right)C_n^kx^k(1-x)^{n-k}$$

In the said paper it is shown that for every function $f(x)$ satisfying the Lipschitz condition with the exponent α we have

$$\max|B_n(x)-f(x)|=O(1/n^{\alpha/2})$$

To obtain this result M. Kac applies the theory of independent functions, orthogonal functions, *etc.* It is interesting to observe that the method used here for the proof of Theorem 3 leads to a very simple proof of M. Kac's theorem as well.

In fact, if

$$|f(x'')-f(x')\leqslant M|x''-$$

then

$$|B_n(x)-f(x)|\leqslant M\sum_{k=0}^{n}\left|\frac{k}{n}-x\right|^\alpha C_n^kx^k(1-x)^{n-k}$$

Using Hölder's inequality and the estimate

$$\sum_{k=0}^{n}\left(\frac{k}{n}-x\right)^2 C_n^k x^k (1-x)^{n-k} \leqslant \frac{1}{4n}$$

we find

$$|B_n(x)-f(x)| \leqslant \frac{M}{2^a n^{a/2}}$$

Later (2), M. Kac observed that his theorem was a special case of an earlier theorem by T. Popoviciu (3) stating that

$$|B_n(x)-f(x)| \leqslant \frac{3}{2}\,\omega\left(\frac{1}{\sqrt{n}}\right) \qquad (5)$$

and gives the proof of the latter theorem according to his own method operating with the theory of independent functions. I have had no possibility of studying the original paper by T. Popoviciu, but the method used in the proof of my Theorem 2 allows stating the Popoviciu theorem in a considerably simpler way than M. Kac's.

In fact,

$$|B_n(x)-f(x)| \leqslant \sum_{k=0}^{n} \omega\left(\left|\frac{k}{n}-x\right|\right) C_n^k x^k (1-x)^{n-k}$$

Hence

$$|B_n(x)-f(x)| \leqslant \omega\left(\frac{1}{\sqrt{n}}\right)\left\{\sqrt{n}\sum_{k=0}^{n}\left|\frac{k}{n}-x\right| C_n^k x^k (1-x)^{n-k}+1\right\}$$

The Buniakowski-Schwarz inequality gives

$$\sum_{k=0}^{n}\left|\frac{k}{n}-x\right| C_n^k x^k (1-x)^{n-k} \leqslant \sqrt{\sum_{k=0}^{n}\left(\frac{k}{n}-x\right)^2 C_n^k x^k (1-x)^{n-k}} \leqslant \frac{1}{2\sqrt{n}}$$

from which (5) follows.

Altay Machine-Building Institute.

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5.III.1944.

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MATHEMATICS

ON A COMPLETE ORTHONORMAL SYSTEM OF THE SPACE L_2 (0,1)

By N. P. ROMANOV

(Communicated by S. L. Soboleff, Member of the Academy, 25.III.1944)

Modifying my method for construction of orthonormal systems, P. S. Hagleev has obtained the following result: if $F(x)$ is defined as a periodic function with period 1, given on $(-\infty, \infty)$ by

$$F(x) = \begin{cases} 1 \text{ if } & 0 \leqslant x < 1/2 \\ -1 \text{ if } & 1/2 < x \leqslant 1 \end{cases}$$

and if $\tau_n(x) = nF(nx)$ for $n = 1, 3, 5, 7, \ldots$, then the orthonormalization of $\tau^1(x)$, $\tau_3(x), \ldots$ leads to the orthogonal system

$$\rho_1(x), \ \rho_3(x), \ \rho_5(x), \ldots$$

where

$$\rho_n(x) = \frac{1}{\sqrt{\varphi_2(n)}} \sum_{d/n} \mu\left(\frac{n}{d}\right) \tau_d(x)$$

$$\varphi_2(n) = \sum_{d/n} \mu\left(\frac{n}{d}\right) d^2 = n^2 \prod \left(1 - \frac{1}{p^2}\right)$$

and where $\tau_n(x)$ and $\rho_n(x)$ are defined only for odd values of n. Such mode of enumeration of $\tau_n(x)$ and $\rho_n(x)$ has shown itself as the most convenient and leading to the most simple and symmetric formulae.

He has shown further that

$$\rho_n(x) = \frac{\varphi(n)}{\sqrt{\varphi_2(n)}} + \frac{2}{\sqrt{\varphi_2(n)}} \sum_{d/n} \varphi\left(\frac{n}{d}\right) d \{2\varphi(dx, d) - \varphi(2dx, d)\}$$

where $\varphi(m, n)$ denotes the number of integers not surpassing m and relatively prime to n.

It is interesting that

$$\sqrt{\varphi_2(n)}\, \rho_n(x) = a_n(x) = \varphi(n) + 2 \sum_{d/n} \varphi\left(\frac{n}{d}\right) \{2\varphi(dx, d) - \varphi(2dx, d)\} d$$

has only integral values and that orthogonality relations

$$\int_0^1 a_n(x)\, a_m(x)\, dx = \begin{cases} \varphi_2^r(n) & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}$$

are expressing some new and untrivial relations, connecting the values of generalized Euler's functions with each other.

We can restate these identities in a non-integral form, taking into account that $\varphi(nx, n)$ are piece-wise constant.

I have found that the cited system is a part of a complete orthonormal system, consisting of functions

$$1;\ \rho_{n;\nu}(x) = \rho_n(2^\nu x);\ \sigma_{n;\nu}(x) = \rho_n\left(2^\nu x + \frac{1}{4}\right)$$

$$(n = 1, 3, 5, \ldots;\ \nu = 0, 1, 2, 3, \ldots)$$

Parceval identities

$$\int_0^1 f(x)^2\, dx = \left(\int_0^1 f(x)\, dx\right)^2 + \sum_{n;\nu} \left(\int_0^1 f(x)\, \rho_{n;\nu}(x)\, dx\right)^2 +$$

$$+ \sum_{n;\nu} \left(\int_0^1 f(x)\, \sigma_{n;\nu}(x)\, dx\right)^2$$

where $f(x)$ is an arbitrary function of $L_2(0, 1)$, and where n runs through all odd positive values and ν through all non-negative values, lead to many untrivial arithmetical identities.

The functions $\rho_{n;\nu}(x)$ are generating in $L_2(0, 1)$ a Hilbert subspace consisting of all functions satisfying almost everywhere the relations $f(x) = -f(1-x)$, and the functions 1, $\sigma_{n;\nu}(x)$ yield the space of functions for which $f(x) = f(1-x)$.

It is interesting to note that $\rho_{1;\nu}(x)$ $(\nu = 0, 1, 2, \ldots)$ are forming the Rademacher's system, the most natural extension of which is our complete orthonormal system.

Tomsk Physical-Technical Institute.

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ASTRONOMY

THE ORBIT OF THE ASTEROID 1036 GANYMED

II. THE DETERMINATION OF THE DEFINITIVE ORBIT

By I. I. PUTILIN

(Communicated by G. A. Shajn, Member of the Academy, 2.IV.1944)

1. Introduction. In the first part of this investigation ([1], [2]) there were given the results of the improvement of the orbit of the asteroid 1036 Ganymed, in which 892 observations of the oppositions 1924, 1926, 1927 and 1928 were used, and the perturbations due to Jupiter, Saturn, Earth and Mars considered. The obtained elements are:

$$
\begin{array}{lll}
\text{Ep. and Osc. 1924. Dec. 14.0 [2424134.0]} & & \\
M & 17^\circ\ 20'\ 42''.43 & \\
\omega & 130\ \ 36\ \ 51\ .61 & \left.\begin{array}{l}\\\\\\\end{array}\right\}\ \text{Ecliptic 1925.0} \\
\Omega & 216.\ 27\ \ 25\ .08 & \\
i & 26\ \ 8\ \ 47\ .57 & \\
\varphi & 32\ \ 37\ \ 16\ .60 & \\
\mu & 815''.23754 & \\
\lg a & 0.4258149 &
\end{array}
\qquad (D)
$$

These elements are very exact. They represent very well all the observations made up to the present time. The representations of the normal places in the sense O—C are shown in the table 4A.

This second part gives the results of determining the definitive orbit of the same asteroid from the 12 oppositions 1924—1938.

2. Observations. The asteroid 1036 Ganymed has been observed during all the oppositions after 1924. Ganymed in some of these oppositions was very distant from the Earth and thus very faint and difficult to observe. But Prof.G. Van Biesbroeck (Williams Bay) kindly observed at my request the asteroid during all these oppositions. I avail myself of the opportunity to express my sincere gratitude for his kindness.

Ganymed was observed during the opposition 1937—38 under very good conditions. It was moving for several months near the Earth. Such conditions are being repeated every 13 years. The ephemerides were then computed by us and published ([3]). Thus, 128 observations were obtained at 16 observatories at that time.

So the total number of observations from 12 oppositions 1924—1938 should be equal to 1052. The Table 1 gives the list of all the observations of Ganymed from these oppositions.

3. Formation of Normal Places. The places of comparison stars were revised as far as possible. The normal places VIII—XIV were formed in the usual manner (for the formation of normal places I—VII see part 1, p. 1262).

Table 1

Nos.	Observatory, Observer	Number of observations	Authority
	Opposition 1924		
1—34	Various observers	873	See 1st part of this investigation
	Oppositions 1926, 1927 and 1928		
35—39	Various observers	19	
	Opposition 1930		
40	Williams Bay, Van Biesbroeck	2	Commun. by letter
41	Johannesburg, Jackson	3	» »
	Opposition 1931		
42	Williams Bay, Van Biesbroeck	3	» »
	Opposition 1932		
43	Williams Bay, Van Biesbroeck	2	» »
	Opposition 1933		
44	Vienna, Krumpholtz	2	» »
45	Algiers, Boyer	1	» »
46	Bergedorf, Schorr	2	» »
47	Johannesburg, Jackson	1	» »
	Opposition 1935		
48	Bergedorf, Schorr	4	» »
49	Johannesburg, Jackson	1	» »
	Opposition 1936		
50	La Plata, Dawson	2	R. I. 1373
51	Bergedorf, Schorr	2	R. I. 1376
52	Willams Bay, Van Biesbroeck	1	Commun. by letter
	Opposition 1937		
53	Algiers, Boyer	9	» »
54	Barcelona, Comas-Sola	1	R. I. 1651
55	Bergedorf, Wachmann	5	R. I. 1640, 1658
56	Bordeaux	1	R. I. 1881
57	Bucarest, Dramba, Popovici	16	Commun. by letter
58	Kiev, Tsherny	3	—
59	Mt. Hamilton, Jeffers	4	Lick. Obs. Bull. 493
60	Nice, Patry	1	Commun. by letter
61	Santiago Ch., Grandon	32	A. N. 266, 231
62	Simeis, Neujmin	5	Commun. by letter
63	Poznan, Kwiek	6	» »
64	Torino (Pino Torineso), Fresa	17	A. N. 265, 20, 313
65	Tokyo, Hirose	8	Letter from Prof. Stracke
66	Uccle, Delporte	2	Commun. by letter
67	Vienna, Krumpholtz	12	Letter from Prof. Stracke
68	West-Coker, Jeovil, Kellaway	6	Commun. by letter
	Opposition 1940		
69	McDonald, Van Biesbroeck	2	» »
	Opposition 1941		
70	Simeis, Neujmin	4	» »

The following procedure was applied for the formation of the normal places of the opposition 1937—38. The interpolation curves were constructed from the differences Obs.—Ephem. for each normal place, using the formula

$$\Delta k = a + b\tau + c\tau^2 + [d\tau^3] \quad (k = \alpha \text{ or } \delta) \qquad (1)$$

where Δk = Obs. — Ephem.; $\tau = t - t_0$, the interval of the time of observation from any initial moment t_0; a, b, c, d, the coefficients which were obtained by means of the least square solution. The weights of observations were obtained according to the Table IV of the first part of this investigation. The normal corrections of ephemerides were found from these curves and afterwards the normal places were formed.

Table 2 gives these normal places. Table 4A gives their representations by the perturbed ephemerides.

Table 2

No.	Greenwich mean time of Universal time	α	δ	Equin.	Number of observations for α	Number of observations for δ
I	1924 Nov. 12.5	20h40m28s.422	+ 3°24′43″.22	1925.0	389	387
II	Dec. 16.5	0 41 27 .936	— 4 49 14 .86	»	228	225
III	1925 Jan. 18.0	2 1 19 .076	— 3 45 14 .28	»	135	135
IV	Febr. 25.0	3 19 57 .077	+ 0 15 52 .65	»	100	100
V	1926 Febr. 27.0	8 47 33 .70	—16 29 44 .0	»	5	5
VI	1927 March 15.0	11 23 36 .60	—20 13 35 .5	»	5	7
VII	1928 May 14.0	14 22 59 .13	— 9 11 50 .5	»	7	7
VIII	1930 Jan. 30.0	7 48 24 .45	—16 30 21 .5	»	4	5
IX	1931 March 3.0	10 36 19 .04	—20 4 27 .1	»	3	3
X	1932 Apr. 21.0	12 58 13 .66	—16 11 48 .5	»	2	2
XI	1933 Sept. 27.0	6 35 11 .09	+ 9 21 42 .4	»	2	2
XII	1934 Jan. 16.0	5 46 54 .18	—13 1 45 .7	»	4	4
XIII	1035 Febr. 23.0	9 46 2 .09	—19 7 56 .3	»	5	5
XIV	1936 Apr. 9.0	11 56 19 .97	—17 47 58 .8	»	5	5
XV	1937 June 16.0	17 14 38 .20	+18 9 13 .3	»	15	17
XVI	Nov. 4.0	20 36 18 .84	+ 5 25 12 .7	»	70	70
XVII	Dec. 29.0	0 22 44 .56	— 3 42 10 .3	»	11	11
XVIII	1938 Jan. 29.0	1 52 20 .06	— 1 29 22 .7	1950.0	12	12
XIV	March 1.0	3 5 10 .50	+ 1 43 5 .6	»	5	5
XX	1940 Apr. 4.0	11 2 32 .17	—17 27 14 .9	»	2	2
XXI	1941 May 11.0	14 11 23 .25	—10 38 36 .6	»	4	4

4. Perturbations. The classical method of variations of arbitrary constants was applied for computing the perturbations. The perturbations due to Jupiter, Saturn, Mars and Earth were taken into account. They were computed according to the formulae given in the first part of this investigation*. The values of the perturbations for epochs of normal places are given in the Table 3. δM is the total perturbation of the mean anomaly.

5. Obtaining the Definitive Elements. In the computation of the definitive elements of the orbit of Ganymed there was applied the method of variation of elements, namely in the form of Bauschinger (1st method) (4). The set of elements (D) was used as elements of departure for computation of the definitive orbit. Normal places I—XIX from the year 1924—38 were used for this purpose. The solution of 38 equations of condition by means of the least square solution gives the following corrections to the elements of departure with their mean errors:

$$\begin{aligned} \Delta M &= -2''.26 \pm 0''.86 \\ \Delta\omega &= +2.74 \pm 0.42 \\ \Delta\Omega &= +2.28 \pm 0.33 \\ \Delta i &= +2.03 \pm 0.32 \\ \Delta\varphi &= +0.14 \pm 0.12 \\ \Delta\mu &= -0.00420 \pm 0.00016 \end{aligned} \qquad (2)$$

* We call to attention a misprint in the first part of our work; the second member of the formula for $d\ \delta e/dt$ should read: $(1 + r/p) \sin \varphi \cdot S_1 \cdot \operatorname{ctg} \varphi$.

These corrections are small. The definitive elements referred to two different epochs are:

Ep. and Osc. 1924 Dec. 14.0 [2424134.0]		Ep. and Osc. 1938 Jan. 2.0 [2428900.5]	
M_0	17°20′40″.17	M_0	18°00′ 6″.85
ω	130 36 54 .35	ω	130 33 2 .93
Ω	216 27 27 .36 (Ecliptic 1925.0) (E)	Ω	216 42 29 .29 (Ecliptic 1950.0) (E′)
i	26 8 49 .60	i	26 9 46 .41
φ	32 37 16 .74	φ	32 40 48 .74
μ	815″.23334	μ	816″.14965
lg a	0.4258164	lg a	0.4254912

Table 4B gives the residuals of the normal places by these elements. Table 5 shows the representations of the normal places XX (1940) and XXI (1941), which were not used in the determination of the definitive orbit.

Table 3

Normal places	Perturbations					
	δM	$\delta\omega$	$\delta\Omega$	δi	$\delta\varphi$	$\delta\mu$
I	+1″.048	—1″.292	+0″.069	+0″.159	— 0″.844	—0″.01221
II	—0 .139	+0 .204	—0 .043	—0 .065	+ 0 .136	—0 .00218
III	—2 .656	+2 .579	—0 .856	—0 .950	+ 1 .633	—0 .02125
IV	—8 .107	+6 .298	—2 .436	—1 .845	+ 4 .488	—0 .04428
V	—2′ 22″.60	+1′ 19″.16	—1′ 12″.13	—3 .37	+40 .32	—0 .15554
VI	—6 16 .28	+3 42 .76	—2 48 .06	+24 .27	+1′ 11″.81	—0 .10269
VII	—9 58 .53	+5 48 .78	—3 19 .79	+52 .54	+1 2 .94	+0 .17941
VIII	—4 57 .27	+4 29 .81	—3 56 .68	+44 .72	+1 29 .65	+0 .74686
IX	+7 16 .74	+2 37 .22	—4 51 .20	+52 .40	+2 0 .39	+1 .18326
X	+20 18 .39	+2 1 .42	—5 30 .04	+1′ .16″.94	+3 8 .90	+1 .26461
XI	+31 57 .42	+1 41 .43	—5 36 .65	+1 25 .33	+3 49 .48	+1 .10346
XII	+34 23 .05	+1 21 .75	—5 34 .80	+1 26 .20	+3 44 .93	+1 .19671
XIII	+46 17 .61	+0 5 .49	—5 36 .34	+1 26 .85	+3 31 .73	+1 .44935
XIV	+1° 0′18″.14	—1 17 .94	—5 40 .36	+1 28 .20	+3 31 .62	+1 .42296
XV	+1 12 59 .75	—3 48 .36	—5 52 .31	+1 5 .15	+3 42 .59	+0 .84171
XVI	+1 15 3 .46	—3 38 .99	—6 10 .56	+1 5 .53	+3 34 .04	+0 .90232
XVII	+1 15 52 .99	—3 34 .35	—6 10 .74	+1 5 .64	+3 32 .11	+0 .91729
XVIII	+1 16 19 .86	—3 29 .73	—6 11 .04	+1 5 .24	+3 32 .08	+0 .90712
XIX	+1 16 44 .84	—3 23 .32	—6 12 .19	+1 4 .68	+3 32 .39	+0 .89092

Table 4

Normal places	A O — C		B O — C	
	$\Delta\alpha$	$\Delta\delta$	$\Delta\alpha$	$\Delta\delta$
I	—0ˢ.01	— 0″.1	—0ˢ.06	+0″.1
II	+0 .01	+ 0 .4	+0 .10	+0 .2
III	—0 .01	+ 0 .2	+0 .01	+0 .8
IV	+0 .01	+ 0 .4	—0 .04	+0 .8
V	+0 .09	+ 0 .4	—0 .20	—0 .1
VI	—0 .10	— 1 .2	—0 .38	—0 .3
VII	+0 .03	— 1 .2	—0 .43	+0 .4
VIII	+0 .41	+ 0 .5	—0 .17	+2 .8
IX	+0 .25	— 2 .4	+0 .19	+1 .7
X	+0 .48	— 0 .7	+0 .13	+1 .2
XI	+0 .51	— 4 .5	+0 .32	—3 .7
XII	+0 .47	+ 0 .4	+0 .39	—2 .0
XIII	—0 .07	— 2 .5	+0 .06	+1 .9
XIV	—0 .04	— 3 .0	—0 .02	—0 .4
XV	—2 .62	—17 .9	—0 .15	+2 .4
XVI	—3 .39	— 7 .6	+0 .12	+0 .6
XVII	—3 .12	+ 0 .7	+0 .03	—0 .3
XVIII	—2 .14	+ 2 .7	+0 .03	+3 .1
XIX	—1 .60	+ 2 .8	—0 .11	+2 .0

Table 5

Normal places	Opposition	$\Delta\alpha$	$\Delta\delta$
XX	1940	$-0^s.12$	$+2''.5$
XXI	1941	$+0\ .08$	$-3\ .0$

The small residuals and the small value:

$$\sum pv^2 = 29''.6 \tag{3}$$

demonstrate the exactness of the obtained elements.

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PHYSICS

SCREENING OF EFFECTIVE CROSS-SECTIONS FOR BREMSESTRAHLUNG AND FORMATION OF PAIRS BY MEANS OF EXPERIMENTAL VALUES OF ATOMFORMFACTOR

By A. KIRPICHEV and I. POMERANCHUK

(Communicated by I. V. Kourchatov, Member of the Academy, 25.III.1944)

In the case of high energies Bremsestrahlung and the formation of pairs are known ([1],[2]) to take place at ever increasing distances from the nucleus. Then screening of the nuclear field by atomic electrons becomes an important problem. Owing to it, the logarithmic growth of effective cross-sections with energy is no longer observed. The limiting value of effective cross-sections corresponding to the so-called complete screening, is usually obtained by Thomas-Fermi method.

However, in the case of light elements with small number of electrons (Z) the use of this method can be legalized by no rigorous proof. It is therefore desirable to find a more accurate value for the effective cross-sections, if Z are small. As has been pointed out in ([3]), the effective cross-sections for Bremsestrahlung σ_r and the formation of pairs σ_p are representable in a simple manner through the atomformfactor:

$$d\sigma_r = 4\alpha Z^2 \left(\frac{e^2}{mc^2}\right)^2 \frac{dK}{KE_0^2} \left\{ \left[E_0^2 + (E_0 - K)^2 - \right.\right.$$

$$\left.\left. - \frac{2}{3} E_0 (E_0 - K) \right] \left[\int_0^{\mu} |1 - F(q)|^2 \frac{dq}{q} + 1 \right] + \frac{E_0 (E_0 - K)}{9} \right\} \qquad (1)$$

$$d\sigma_p = 4\alpha Z^2 \left(\frac{e^2}{mc^2}\right)^2 \frac{dE}{E_0^3} \left\{ \left[E_0^2 + (E_0 - E)^2 + \right.\right.$$

$$\left.\left. + \frac{2}{3} E_0 (E_0 - E) \right] \left[\int_0^{\mu} |1 - F(q)|^2 \frac{dq}{q} + 1 \right] - \frac{E_0 (E_0 - E)}{9} \right\} \qquad (2)$$

where $\alpha = \frac{e^2}{\hbar c}$, $F(q) = \frac{1}{Z} \int_0^{\mu} \rho(r)\, e^{i \vec{q} \vec{r}/\hbar c}\, d\tau$; Ze is the charge of the nucleus; E_0 in (1) is the electron's energy previous to emission; E, after emission; K is the energy of the radiated quantum; E_0 in (2) is the energy of the photon forming the pair; E is the energy of one of the components of the pair; ρ is the density of the charge in the atom; $d\tau$, a volume element.

By means of (1) and (2) the effective cross-sections can be computed from the experimental values of the atomformfactor, which are known, *e. g.*, from the scattering of X-rays.

It should be noted at once that the difference between the effective cross-sections obtained in this way and those obtained by Thomas-Fermi method is not great; its order of magnitude is 7 — 8 per cent. In the majority of problems

where the values of σ_r and σ_p are of importance, especially in most of the cascade-theoretical problems([4],[5]), the difference obtained by us does not affect the order of magnitude of the derived results. There exists, however, a region of phenomena connected with large cascade-showers (Auger showers) ([6—8]) for which the question as to the thickness of the atmosphere in radiation units ([3]) plays a very important rôle ([9]). When the effective section (and, accordingly, the radiation unit of length) changes by as much as 8 per cent, the thickness of the atmosphere changes by 2.1 units*.

Such a change results, for instance, in that the density of the particles in a shower created by primary electrons and having a maximum at the altitude of Jungfrau changes as much as $e^{1.4}=4$ times. This result is of great importance for the theory of coincidences between counters, due to Auger showers.

Making use of the data on atomformfactor referred to in ([11]), we obtain the following values for the radiation logarithm**

$$L_r=\int_0^{\mu}|A-F(q)|^2\frac{dq}{q}+1$$

Z	8	9	12	13	14	17	18	19	20	26
L_r from exper. data	.954	4.85	4.71	4.68	4.65	4.65	4.75	4.64	4.62	4.55
L_r according to Thomas-Fermi	4.56	4.52	4.42	4.40	4.37	4.31	4.45	4.27	4,25	4.17

It may be seen from this table that the method of Thomas-Fermi gives an underestimated value for the effective cross-sections. The underestimation is 6—8 per cent on the average. The radiation units (t-units) of length are, as a matter of fact, lower than those obtained according to Thomas-Fermi.

	Water	13 Aluminium	18 Argon	26 Iron
t-unit from exper. data (g/cm²)	35	22.4	19.4	13.3
t-unit according to Thomas-Fermi (g/cm²)	38	23.9	20.8	14.4

Although the data for nitrogen were not treated, the effect under consideration will not be lower for nitrogen than for oxygen. From this it may be concluded that in the air, the t-unit is 9—10 per cent less than is usually adopted. In other words, it is equal to 35 cm water equivalent instead of 38.

The change of t-units results in the respective change of the critical energy of the cascade theory.

In conclusion we wish to express our appreciation to Prof. L. Landau for his interest in the problem.

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* The ordinary value for the thickness of the atmosphere is 26 t ([10]).

** According to Thomas-Fermi, $L_r=\ln 191\ Z^{-1/3}$.

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PHYSICS

REMOVAL OF LATENT IMAGE OF α-PARTICLE TRACKS FROM A PHOTOGRAPHIC LAYER

II. RELATIONSHIP BETWEEN THE DEGREE OF EXCITATION OF A PHOTOGRAPHIC EMULSION GRAIN AND ITS DEVELOPMENT

By N. A. PERFILOV

(Communicated by V. G. Chlopin, Member of the Academy, 20.IV.1944)

In an earlier work (1) the author has shown that the excited grains of a photographic emulsion will not develop after the latent image of α-particle tracks has been acted upon by certain oxidizers. However, the exposures in the described experiments were so short that it was highly improbable that the excitation of a grain might exceed that which is obtained when the grain had been crossed only once by an α-particle. In the present paper the author describes his experiments on the action of $K_3Fe(CN)_6$ upon a latent image when the exposure to α-particles is as long as to secure a repeated excitation of the emulsion grain.

Suppose that during an interval of t seconds, N α-particles fall upon a unit surface of the photographic plate containing m grains, and that every α-particle causes such a state of excitation where a selective development of k grains is possible.

Under these conditions an average grain is acted upon $kN/m = \bar{n}$ times. The probability that an individual grain will be excited n times is then expressed by the well-known formula of Poisson

$$P(n) = \frac{(\bar{n})^n e^{-\bar{n}}}{n!} \qquad (1)$$

That this formula applies to photographic processes in the case of visible light, has been proved by several different authors(2), and there is not the slightest reason to doubt that it should also be valid when the falling beam of rays consists of charged atoms.

Moreover, in the case under consideration the «critical number» is equal to unity, which follows from the fact that tracks of separate α-particles are observable in the emulsion so that the applicability of the above formula can easily be verified by experiment. The author has made this verification by comparing the curves computed according to the formula with those obtained experimentally, by plotting the density of the developed grains against the exposure time.

The total number of grains excited n times is obviously equal to $mP(n)$. We shall be interested in the total number of grains excited q times at the least, and have therefore to sum up the last results over all values of n from q to infinity. Denoting the sought-for quantity by y, we obtain:

$$y = m \sum_{n=q}^{\infty} P(n) = m \left[\sum_{n=0}^{\infty} P(n) - \sum_{n=0}^{q-1} P(n) \right]$$

Taking notice of the fact that

$$\sum_{n=0}^{\infty} P(n) = 1$$

we divide the equality by m to obtain the formula

$$\frac{y}{m} = 1 - \sum_{n=0}^{q-1} \frac{(\bar{n})^n e^{-\bar{n}}}{n!} \qquad (2)$$

which is very convenient for calculations.

To represent this relation graphically for different n, we plot the values of y/m against q and obtain a family of curves whose points of intersection with the straight line $q=a$ will give a number of grains excited no less than a times for all n.

Or, proceeding the other way round, suppose that by acting on the latent image for different n we take off some portion of the full excitation and then count up the number of grains which are still capable of development, then, laying off the obtained values on the respective curves, we obtain the straight line $q=a$. The existence of this straight line obtained by experiment is incontrovertible evidence that irrespective of the length of exposure the action of oxidizers upon the latent image under certain conditions (concentration of the oxidizer, time of its action) results in only those grains becoming incapable of development for which $q<a$. For example, the grains that have been excited by a single passage of α-particles can no longer be developed, whereas the grains excited by two, three or more passages will develop normally, as if they were not acted upon in the interval between exposure and development.

Experiments and Discussion of Results. In order to be able to count the developed grains under any length of exposure, it was necessary to have plates with a suitable density of grains. For this purpose 1 cm³ of a fine-grained emulsion capable to record the paths of individual α-particles was diluted up to 100 cm³ by 3 per cent gelatine melt. A strictly measured quantity of the emulsion, such as gave a sufficiently thin layer, was poured over 9×12 glass plates.

The emulsion was allowed to turn to a jelly on a strictly horizontal surface, and then dried.

A photographic plate thus prepared was divided into squares whose number corresponded to the intended number of exposures, and then each cell was exposed to α-particles.

The time of exposure was doubled from one square to the next.

The source of α-particles was a silver disk 10 mm in diameter coated with electrolytically deposited polonium and placed for convenience in a special frame. In order to eliminate the influence of luminescence in the layer of air between the source and the plate whose thickness was 1 mm, the source was covered by an aluminium foil, about 5μ thick. The intensity of the source corresponded to the formation in the photolayer of a normal concentration which was about 10^5 tracks of α-particles per minute per 1 mm² of area. The irradiated plate was placed for 15 minutes in a water solution of a substance capable to destroy the latent image; then it was washed, dried and developed. By preliminary experiments the time of development was selected to ensure a sufficiently small background.

The composition of the developing solution was the same as in the previous experiments*.

All experiments were run under strictly constant conditions. After the development several photomicrographs were taken of every square at the point

* Water, 110 cm³; crystalline sulphide, 15 g; hydroquinone, 1.5 g; potassium bromide, 0.5 g; potassium hydroxide, 0.5 g.

corresponding to the centre of the polonium source. From the negatives magnified positives were prepared which served for counting the grains. The positives had a linear scale which permitted with any magnification easily to determine the respective area.

In order to plot y/m against q one must know m and $\bar{n}$. The first was determined by counting the grains on photomicrographs taken from the undeveloped plate. In order to determine $\bar{n}$, several short exposures were made so as to obviate errors in counting twice excited grains, then the number of grains which had acquired the developing power after a unit time exposure was counted. From the value thus obtained n was determined for the exposures used in the experiment, and the relation between y/m and q was calculated by means of formula (2).

The results of the experiments with diluted emulsions are shown in Fig. 1. The points marked off on the ordinate axis (one time excitation by α-particles)

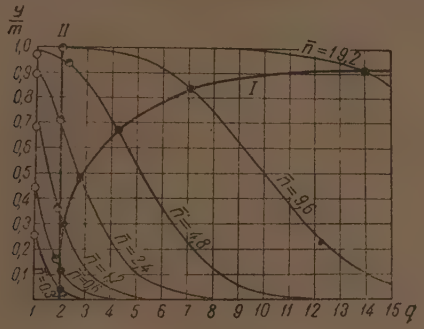

Fig. 1

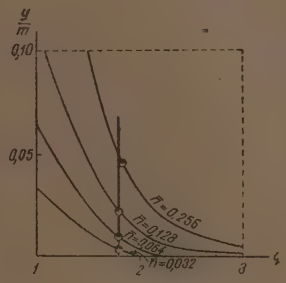

Fig. 2

are the result of counting the developed grains on the control plate. These data agree fairly well with those calculated according to formula (2). Curve I refers to a plate which after exposure and previous to development was bathed in a water solution of $K_3Fe(CN)_6$, concentration 10^{-4}. It may be seen that different values of q are obtained from different exposures, q growing with n. The quantity q does not cease growing even with those values of n for which the excitation of all grains is ensured by no less than two passages of α-particles. To explain this run of the curve we may suppose that upon the action of $K_3Fe(CN)_6$ on the latent image the silver atoms composing the image are oxidized to a compound insoluble in water and sparingly soluble in the developer, and this compound ($Ag_3Fe(CN)_6$) screens the centres of development from the action of the developing solution.

In the next experiments the $K_3Fe(CN)_6$ solution was added by an equivalent amount (1.2×10^{-4}) of hyposulphide* in order to prevent the formation of a screening layer. Otherwise the conditions of the experiment were the same. In this case all the experimental points fit within a straight line (curve II, Fig. 1), corresponding to $q\cong2$ for all values of $\bar{n}$. It will be obvious that under conditions preventing the formation of a screening layer, the phenomenon of annihilation of the latent image appears in its pure state. The grains excited by a single passage of α-particles lose their ability of development, while the grains excited two times or more develop perfectly as if they had suffered no influence between the exposure and the development. This result, as will be seen below, may have an important practical application.

* Silver bromide can also be dissolved by hyposulphide, but the concentration of the latter is not sufficient for changing the grain size in the emulsion appreciably during the time of bathing. Two photoplates exposed for the same time and afterwards put for 15 min. one in a solution of hyposulphide of the above-mentioned concentration, and the other in distilled water, gave upon development an equal number of grains per unit area.

In Fig. 2 are represented the results obtained by repeating the experiment with photographic plates which this time were covered by an undiluted emulsion. In such normal plates the counting of grains after development is possible only with short exposures. The direction of the experimental straight line gives ground to suppose that the process had the same trend as in the case of diluted emulsions. The position of the curve somewhat to the left of $q=2$ can be explained by that the conditions for the penetration of the oxidizing solution into the layer are in this case unfavourable because of the high concentration of silver bromide grains. To obtain from the oxidizer the same effect, the bathing must be prolonged compared to the time used with diluted emulsions.

The conclusion to be made from the results obtained is as follows. When the latent image produced by the passage of α-particles through the photographic layer is acted upon by the oxidizer $K_2Fe(CN)_6$ in the presence of a substance able to dissolve the oxidation product, it is possible to destroy the development centres due to a single excitation by α-particles, and these grains lose their ability of selective development. Whereas the grains excited two, three or more times will develop normally.

This suggests the following utilization of our results in practice. If we have a radiation consisting of α-particles and of other particles losing more energy per unit path, then by recording such a flow of particles by means of a photographic plate and by treating then the photographic layer in the above-mentioned way, we can remove the excitation from the grains excited by α-particles, and only the tracks of those particles will develop which suffer greater specific losses of energy. This has been confirmed by the experiments (the results of which will be published later) in which uranium fragments were recorded without the background of α-particles emitted by uranium.

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TECHNICAL PHYSICS

STRENGTH OF AMORPHOUS AND OF CRYSTALLIZING RUBBER-LIKE POLYMERS

By A. P. ALEXANDROV, Corresponding Member of the Academy, and J. S. LAZURKIN

The strength of unloaded rubbers and the action of active fillers are very different for rubbers derived from different caoutchoucs.

This difference is not explicitly connected with the chemical composition of caoutchouc. Thus, for example, rubbers of natural and synthetic isoprene caoutchouc differ with respect to strength in the ratio 10—15 to 1, whereas rubbers of butyl caoutchouc and of chloroprene caoutchouc are very similar to natural ones with respect to strength.

The difference in strength cannot also be ascribed directly to the difference in structure of the chains, linear or branched; linear polymers of styrol and of methyl-metacrylate «vulcanized» by small admixtures of butadiene-benzene or dimeta-crylate-ethylene-glycol, possess in the elastic state strengths of just as small magnitude as unloaded rubbers of sodium-butadiene caoutchouc, or of isoprene caoutchouc, *etc.*

The difference in strength must accordingly be looked for in the different macroscopic properties of these polymers.

Table 1

Kind of caoutchouc	Tensile strength of unloaded rubber (kg/cm²)	Tensile strength of loaded rubber (kg/cm²)	Tensile strengthening influence of the active fillers
A. Rubbers of the I group			
Butadiene of sodium polymerization	15—25	120—180	from 5 to 12 times
Butadiene	15—25	100—150	from 4 to 10 times
Buna S	20—30	150—200	from 5 to 10 times
Buna N	20—40	150—200	from 4 to 10 times
Isoprene of sodium polymerization	15—20	100—150	from 5 to 10 times
Methyl caoutchouc	20	80—120	from 4 to 6 times
B. Rubbers of the II group			
Natural caoutchouc	200—300	300—325	from 1 to 1.6 times
Chloroprene caoutchouc	150—200	200—250	from 1 to 1.7 times
Butyl caoutchouc	250—350	300—350	from 1 to 1.4 times

Remark. In the table the highest authentic values of the strength are given, which sometimes have been obtained on mixtures deprived of technical importance. The loaded rubbers are taken with such a content of the filler which corresponds to the highest value of the strength.

In Table 1 the strength of unloaded and loaded rubbers of different caoutchoucs are compared. It is seen from the table that all the rubbers can be divided into two sharply different groups, the strength of unloaded rubbers of the first group being of the order of 25 kg/cm² and that of unloaded rubbers of the second group of the order of 250 kg/cm².

The strength of rubbers of the first group is increased by the introduction of the filler up to the value characteristic of the rubbers of the second group, while the strength of the latter remains practically unaltered by the loading.

The rupture of rubbers of the I and II group takes place at extensions of the same order of magnitude—600—800 per cent. If the strength of the rubbers of the first and second group is compared under such conditions when the rupture takes place at small extensions of the specimen, that is under wearing out by rubber, fatigue or tearing, the rubbers of the groups are found to behave in a quite similar way.

The large strength of unloaded rubbers of the II group is thus manifested in such a rupture only which is preceded by a large deformation. This seems to indicate that rubbers of the II group are strenghtened by certain macroscopic changes which arise in the process of deformation.

Comparing the caoutchoucs of the I and II groups from this point of view, we see that all the caoutchoucs of the second group and the rubber obtained from them crystallize, while the rubbers of the first group do not crystallize. In the rubbers of the second group a partial crystallization takes place with extension. The small crystals play, according to our opinion, the rôle of an active filler and increase the strength of the rubber.

The unloaded rubbers of the II group, thus, behave in the case of large deformations as «self-loading».

The influence of the crystallization of rubber on the shape of the stress-strain curve $F(L)$ was long ago noted by various investigators (see, for instance, [1]).

In 1940 Field [2] in his studies of the crystallization of natural rubbers vulcanized by various methods, has noted the parallelism between the increase of the amount of the crystalline phase and the strength, but has not referred the increase of the strength to the influence of the crystals, as particles of an active filler.

The particles of crystalline caoutchouc in the rubber are very finely dispersed and very intimately connected with the amorphous phase. The mechanical properties of these crystals are apparently similar to the mechanical properties of paraffin crystals, *i. e.* their proper strength is very small. One cannot therefore tell *a priori* whether such crystals can actually cause a strengthening of the substance.

In order to solve this question the following model experiments were carried out by us: the unloaded rubber of sodium-butadiene caoutchouc was swollen in fused paraffin or stearic acid. The breaking strength of the swollen specimens was therefore measured at a $t°$ of 2—3° above that of fusion point of paraffin or stearic acid.

In this case, as can be seen from Table 2, the strength was very low.

If, however, the extended specimen was cooled 2—3° below the fusion temperature of the paraffin or stearic acid, then at the moment of their crystallization the hardness of the rubber sharply increased, the curve $F(L)$ sharply rose and the breaking took place for stresses by a factor 4—11 higher than in the case of a liquid component, introduced in the process of swelling.

More than that, the tensile strength of the rubber containing the crystalline phase exceeded by a factor 2—3 the strength of the initial rubbers. This model experiment confirmed the possibility of increasing the tensile strength of the rubber at the cost of the crystallizing caoutchouc.

Let us consider in the light of this conception a few other properties of «self-loading» rubbers.

1. Small Influence of Active Fillers on the Tensile Strength of Rubbers of the Second

Table 2

Tensile Strength of Rubber from Sodium-Butadiene Caoutchouc

Initial rubber (g)	Swollen, above the fusion temperature of the introduced component (g)	Swollen, below the fusion temperature of the introduced component (g)	Weight % of the introduced component (with respect to the rubber) and kind of the substance
710	—	—	—
	200	2240	80% stearic acid
	320	1320	85% paraffin
	330	1840	65% palmatic acid

Remark. The tensile strength refers to the specimen. All the initial specimens had the same cross-section and were torn away directly or after swelling. Each figure ot the table represents an average of the values for 5 specimens.

Group. This small influence is due to the fact that near the breaking point, owing to the «self-loading», the rubber contains a number of active fillers—crystals—lying close to the optimal one. Therefore the introduction of an additional

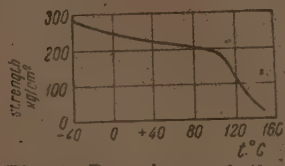

Fig. 1. Dependence of the strength of rubber from natural caoutchouc on the temperature (according to Van Rossem) (3)

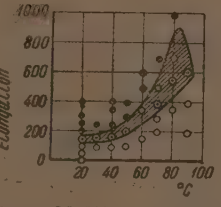

Fig. 2. Melting temperature of crystalline caoutchouc for various extensions (according to Susich) (4). Points, crystalline; small circles, amorphous. Shaded region and circles with points, partially fused caoutchouc

active filler can change the strength in one or the other direction but insignificantly.

2. Sharp Drop of the Strength of the Rubbers of the Second Group in a Narrow Temperature Range. Fig. 1 shows the dependence of the strength of natural rubber on the temperature. At a temperature about 120° the strength decreases sharply—from 260 kg/cm² to 20 kg cm² in the case of an extension till the breaking point of the same order as before, about 800 per cent is reached.

Comparing this curve with fusion temperatures of crystals at different extensions determined by Susich (4) (Fig. 2), we see that in the case of elongations about 800 per cent the fusion of the crystalline phase takes place at temperatures 100—110° C.

Owing to the fusion of the crystalline phase, the strength of the rubber drops from the curve of the «strength of the loaded rubber» to the curve of the «strength of unloaded rubber», as shown in Fig. 1.

3. A Sharp Optimum of Vulcanization of Unloaded Natural Rubber (determined by the tensile strength). The vulcanization decreases the tendency of caoutchouc to crystallize. With increase

of the number of space bridges, formed by sulphur or by some other vulcanizing factor, the critical extension at which the process of self-loading of the rubber by the crystalline phase starts, is increased, while the quantity of the crystalline phase at the breaking point is decreased, which leads to a decrease of the strength. As soon as the critical extension becomes larger than that corresponding to the breaking point, the strength of the rubber sharply decreases (drops to the curve of the «strength of unloaded rubber»).

4. Large Decrease of the Work of Breaking in the Case of Rapid Deformations. The lowering of the work of breaking in the case of rapid deformations, which, according to our measurements, is especially strong in chloroprene rubbers, is connected with the fact that the velocity of the crystallization becomes comparable with the rate of destruction and the specimens are therefore destroyed at smaller contents of the crystalline phase than in the case of a slow breaking.

The mechanism of the strengthening action of active fillers in rubber-like substances consists mainly in the fact that the filler furthers the equalization of the stresses in the specimen.

In an elastic polymer the space macromolecular lattice is built up irregularly, when it is strained, in some of its elements considerable overstresses arise which start the process of breaking, while the major part of the polymeric lattice is either not at all stressed or stressed but slightly.

In a substance with active fillers a considerable part of the space lattice is adsorbed on the surface of the particles of the filler, the corresponding binding being larger than the interaction force of the polymer molecules with each other and smaller than their strength.

Owing to such an intermediate value of the adsorption forces, the magnitude of the overstresses arising in a strained polymeric lattice cannot exceed the adsorption forces. As soon as the overstress in some region reaches this magnitude, the desorption of stressed chain begins, so that its stress does not increase with further deformation. Slightly stressed and non-stressed chains remain sorbed and the stress on them rises.

The total stress of the strained specimen is thus distributed in a much more homogeneous way between the elements of its space lattice, which leads to the increase of the modulus and of the tensile stress. The breaking takes place when the possibility of an equalization of the stresses owing to desorption is exhausted.

The crystals play the same rôle, but the surface sorption is replaced in this case by the packing of polymer chains into a crystal.

As follows from the aforesaid, the high tensile strength of certain unloaded rubbers is due to their «self-loading» by their crystalline phase in the case of extension.

This strength has thus nothing to do with the stability of the rubber against destruction by other kinds of deformation.

It is possible therefore that for the characteristics of the properties of rubbers in articles which suffer but slight deformations, it is quite needless to aim at high tensile strength at large deformations.

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PHYSICAL CHEMISTRY

THE CRYSTAL STRUCTURE OF $Hg(CN)_2$

By G. S. ZHDANOV and E. A. SHUGAM

(Communicated by A. N. Frumkin, Member of the Academy, 16.V.1944)

In this work an attempt was made to establish the structure of mercury cyanide $Hg(CN)_2$. The investigation was based upon the experimental data of Hanawolt [1] and Hassel [2]. The latter has carried out an X-ray investigation of $Hg(CN)_2$ by the rotation method, has found the space group V_d^{12} for this compound and has established the arrangement of the mercury atoms. They are in position $8d$ [3] with the parameter $x_{Hg} = 0.21$. The intensities found by Hanawolt and Hassel agree fairly well (see table, columns 4, 8).

Experimental and Calculated Values of Intensities in $Hg(CN)_2$

kkl	*d* calc.	*I* calc. by Hanawolt	*I* of Hanawolt	*I* calc. after Hg	*I* calc. with (CN)	*I* calc. prob.	*I* of Hassel
1	2	3	4	5	6	7	8
101	6.56	—	0	6	0	2	0
200	4.85	4.84	83	90	87	92	st.
211	3.80	—	0	18	14	4	w.
112	3.75	3.72	100	100	100	100	v. st.
220	3.42	3.42	13	32	14	20	m. st.
202	3.29	—	0	0	0	3	0
310	3.06	—	0	1	1	2	w.
301	3.04	3.03	10	7	14	13	m. st.
103	2.86	2.84	7	1	3	4	—
221	2.57	2.58	10	13	23	13	w.
312	2.53	2.51	67	60	60	66	st.
213	2.45	—	0	5	8	10	m. st.
400	2.42	2.42	13	11	20	13	st.
411	2.26	2.28	10	10	12	12	m. st.
004	2.23	2.22	7	7	4.5	7	m. st.
303	2.19	—	0	3	4	3	w.
420	2.17	2.17	13	13	14	12	st.
402	2.08	—	0	1	1	0	0
332	2.04	2.02	33	12	2	13	w.
204	2.03			20			m. st.
323	1.99			7			w.
422	1.95	—	0	2	2	2.7	—

As the scattering power of the mercuric atoms is much greater than of the cyanide groups, the chief part of the scattering may be attributed to the heavy atoms of the mercury. The calculated intensities for the atoms of mercury agree fairly well with the experimental data; there are, however, certain discrepancies between them (see table, columns 4, 5, 8). Of the lines with small indices, such discrepancies were observed for 101, 220, 211, 213, 103.

We have attempted to do away with these discrepancies in two ways: 1) by making the parameter of the Hg-atoms more precise, 2) by including in our calculations cyanide groups, the position of which may very likely be determined from the above discrepancies. The first way gave no single result for the whole range of lines, but using the second way we were more successful.

We began by determining the coordinates of the centre of the cyanide group from the structure amplitude 220 and 101, from which $x, y, z = 1/4, 0, 1/4$ were found. The checking up with the other lines showed some improvement in the agreement between the experimental and calculated intensities. This led to the supposition that the coordinates of the CN-groups are actually close to the values 1/4, 0, 1/4 (see table, columns 7, 6). From the position established for CN-groups a double coordination of such groups for each Hg-atom could be assumed. In this way the assumption that the lattice of $Hg(CN)_2$ is molecular has been borne out.

When we began the more exact determination of the coordinates, we bore in mind the following considerations and data: for the molecule $Hg(CN)_2$ it is natural to expect the linear structure, for the valence angle for the Hg-atom

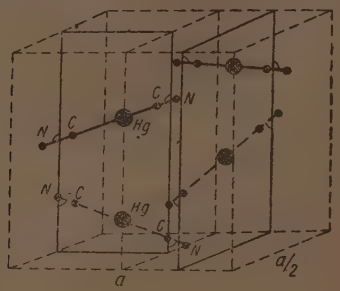

Fig. 1

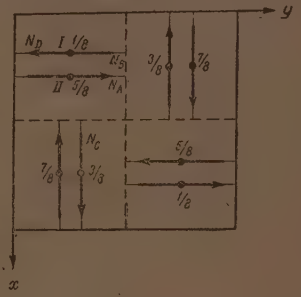

Fig. 2

equals 180°, the distance C—N equals 1.15 Å, the distance Hg—C we assumed to equal 2.2 Å, as in the $Hg(CH_3)_2$ [4], and the Van-der-Vaals distance N—N of the neighbouring molecules being 3 Å. Taking into consideration all the above-mentioned facts, we obtained the following coordinates for the carbon and nitrogen: $x_C = 0.21$; $y_C = 0.03$; $z_C = 0.18$; $x_N = 0.21$; $y_N = -0.08$; $z_N = 0.21$. We calculated the intensities for what we thought to be the most likely model using these coordinates. The results of the calculation are given in the table, column 7.

They showed a slight difference as compared with those obtained for the «unsplit» CN-groups, because of the smallness of the scattering by the carbon and nitrogen atoms. Both calculations agree fairly well with the experiment. It seems difficult to obtain better agreement because of the probable errors in the experimental data.

The proposed structure of $Hg(CN)_2$ is given in Fig. 1 for half of the unit cell. In Fig. 2 is given the projection of the full structure on (001). The figures in Fig. 2 denote the heights of the Hg-atoms, while arrows indicate the cyanide group with the greater height.

The interatomic distances in $Hg(CN)_2$: $Hg\,I - N_C = 2.52$ Å; $N_B N_C = 4.1$Å; $N_C N_D = 4.16$Å; $Hg\,II - N_C = 3.32$Å; $N_A N_B = 6.1$ Å; $N_A N_C = 4.84$ Å. It is very interesting that the distance between Hg and C in the neighbouring molecules (2.52 Å) is smaller than the usual Van-der-Vaals distance. This points to an interaction between them, which may be accounted for by the fact that in some valence of the $Hg(CN)_2$ molecule, the Hg-atom carries a positive charge, while the N is negatively charged.

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CHEMICAL TECHNOLOGY

ON THE THEORY OF SOLIDIFICATION OF SULPHATED SLAG CEMENTS

By P. P. BUDNIKOV, Member of the Ukrainian Academy of Sciences, and V. M. LEZHOEV

The study of the chemism of processes taking place in systems corresponding to sulphated cements has revealed to the authors the possibility of obtaining new kinds of klinkerless(1) and gypsum slag cement (2).

Through studying the changes taking place in the liquid phase of a solidifying slag cement the authors have established that the gypsum whose content in the cement is about 20 per cent, disappears from the liquid phase after 24—48 hours if the blast-furnace slag contains about 20 per cent of alumina, and after 72—144 hours, if the content of alumina is lower (about 10 per cent). If gypsum is present in the liquid phase, no alumina is found in the solution, or else it occurs there in quite insignificant quantities. Alumina appears in the liquid phase only after the disappearance of gypsum. At the same time, there is observed an appreciable increase in the content of calcium oxide in the solution. The authors have found that calcium aluminates are more readily soluble when the concentration of calcium oxide attains a value of 0.4—0.5 g per litre.

When granulated blast-furnace slag and gypsum interact in lime-water (at a ratio corresponding to that of a sulphated slag cement), there are formed compounds given in the table.

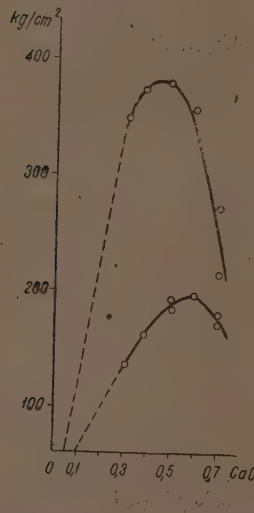
Characteristics of blast-furnace slags			New formations		
Name of slag	Modulus of alkalinity $\frac{CaO+MgO}{SiO_2+Al_2O_3}$	Index of activity $\frac{SiO_2}{Al_2O_3}$	Composition from the data of chemical analysis	Limit concentration, g per litre	
				CaO	CaSO
Basic . . .	1.04	1.27	$3CaO \cdot Al_2O_3 \cdot 3CaCO_4 \cdot aq + 2CaO \cdot Al_2O_3 \cdot aq + 1.5CaO \cdot SiO_2 \cdot aq$	0.03	0.10—0.11
Acid . . .	0.80	1.72	$3CaO \cdot Al_2O_3 \cdot 3CaSO_4 \cdot aq + 1.22CaO \cdot SiO_2 \cdot aq$	0.04	0.11
$CaO \cdot SiO_2 \cdot 2.5H_2O$ (according to Le-Chatelier)				0.052	—
$3CaO \cdot Al_2O_3 \cdot 3CaSO_4 \cdot 30H_2O$ (according to Lafuma)				0.045	0.11

The two slags investigated by the authors were distinguished by a rather high content of alumina (20—24 per cent), and this accounts for the formation of large quantities of sulphoaluminate ($3CaO \cdot Al_2O_3 \cdot 3CaSO_4 \cdot 30H_2O$), alongside of which

a bi-calcium hydroaluminate ($2CaO \cdot Al_2O_3 \cdot 7H_2O$) is also formed, owing to the high ratio of alkalinity in the slag. The possibility of a simultaneous formation of sulphoaluminate and hydroaluminate of calcium is also corroborated by Johns' data (3).

The acid blast-furnace slag investigated by the authors (from the Urals) is remarkable for its readiness to enter into reaction and is therefore a good material for the production of a high quality cement. The main blast-furnace slags from our southern metallurgical works contain a high ratio of silica and 8—12 per cent of alumina. Their index of activity ranges within 3.0—5.0. The interaction of these slags with gypsum in water leads to the formation of comparatively small amounts of sulphoaluminate, at the expense of which there is an increase in the amount of calcium hydrosilicate ($n \cdot CaO \cdot SiO_2 \cdot aq$).

As is known, the formation of calcium sulphoaluminate in portland and slag portland cement is associated with stronger swelling, and if the sulphoaluminate is formed in great quantities, the result is the collapse of the solidified cement. In the case of sulphated slag cement, swelling was not observed unless the cements had an irregular composition, so that there was a high concentration of calcium oxide in the liquid phase.



The intensity of solidifying depends upon the presence in the cement of sufficient quantities of gypsum and upon the concentration of calcium oxide in the solution. In case the content of calcium oxide is low (under 0.4 g per litre), the aluminate of calcium passes into solution in but slight quantities, and the result is a poor formation of calcium sulphoaluminate which unfavourably affects the strength of the cement. If the content of calcium oxide in the solution is high (above 1.08 g per litre), the aluminates of calcium do not dissolve and the cement is one of a swelling kind, owing to the formation of calcium sulphoaluminate in the reaction of solid aluminates of calcium with the dissolved calcium oxide and gypsum.

For a normal solidification of a sulphated slag cement it is necessary that the liquid phase contain 0.4—0.5 g per litre of calcium oxide (with this amount the aluminates dissolve the most readily) and at any rate not above 1.08 g per litre. This condition is secured by adding a little lime (up to 3 per cent) or portland cement (up to 7 per cent) to the mixture of gypsum and slag, because calcium oxide is split off by the slag in very small quantities and is rapidly absorbed for the formation of calcium sulphoaluminate. These considerations are confirmed by the experimentally established dependence of the quality of the cement upon the content of calcium oxide in the liquid phase. The results are represented graphically in the adjoined figure. The lower curve brings out the ultimate compression strength in kilograms per cm² of standard sample after 7 days of solidification in water, the upper curve corresponds to a solidification age of 28 days. The content of calcium oxide was determined 2 hours after the cement was mixed with water.

The dependence of the quality of a sulphated slag cement on the content of calcium oxide in the liquid phase, as established by the author, affords the possibility of properly choosing the composition of the cement.

After the cement is mixed with water, the latter is at once completely saturated with calcium oxide and gypsum. Owing to the high concentration of calcium oxide, the aluminates enter into reaction with it and with gypsum and continue in the solid state, this sets up the conditions for swelling. As, however, the rate at which calcium oxide is bound, when sulphoaluminate and hydrosilicate of calcium are forming, exceeds the rate of its going over into solution, the latter rapidly decreases in calcium oxide and the dissolving aluminates form sulphoaluminate without swelling. The formation of sulphoaluminate comes rather rapidly to an end, and

this completes the initial period of the solidification of the cement. Thereupon, owing to an intense extraction of calcium aluminates, the surface of the cement particles suffers considerable corrosion which leads to a strong development of the reaction of formation of hydrosilicates and hydroaluminates of calcium, and this makes the cement to increase in strength during a considerable period of time.

A further formation of calcium sulphoaluminates until the aluminates of calcium in the cement are used up to the full extent, is possible if the cement solidifies in sulphate waters, particularly in sea water. No swelling of the cement takes place in this case because of the low concentration of calcium oxide. Our investigations have shown that a sulphated slag cement acquires a higher strength in sulphate waters than in fresh water, which is accounted for by the further formation of sulphoaluminates. Sulphated slag cements solidify well in sulphate waters and, unlike other hydraulic cements, are not liable to collapse under these conditions. And the limiting concentration of calcium oxide for new compounds being not very high (0.045 and 0.052 g per litre), the sulphated cement enjoys high hydraulic qualities and, accordingly, serves a long time in hydrotechnical constructions.

The evolution of heat upon the solidification of the sulphated slag cement is about 20—30 calories per 1 g of cement, which is considerably less than the amount of heat given off by a solidifying portland cement (36—50 calories per 1 g). This property speaks in favour of the sulphated slag cement when it is used in massive concrete constructions. But not so in the case of thin walls erected at low temperatures. The heat evolved under these conditions is rapidly dissipated, whereby the process of solidification is damped down or even checked completely. At the same time the solubility of calcium oxide increases and is 1.4 g per litre at 0° C, 1.3 g per litre at 10° C, which sets up conditions for swelling. The result is that the solidified cement has a poor strength. Things are different in the case of high temperatures (40—80° C), when concrete articles are manufactured by the method of electric or steam heating. At these temperatures the cement attains its maximum strength after 24—48 hours. The solubility of calcium oxide decreases with increasing temperature: it is 1.07 g per litre at 40° C, 0.88 g per litre at 60° C, and 0.71 g per litre at 80° C. Therefore no swelling will occur if the temperature is high. In this case it is advisable for increasing the rate of solidification to add somewhat more calcium oxide or portland cement than is usually added at ordinary temperature.

Conclusions. The solidification of sulphated slag cements like that of other hydraulic cements is determined by the molecular forces of cohesion between solid hydrated new formations. The distinguishing feature of sulphate slag cement is that the sulphoaluminated calcium which forms in great quantities on their solidification does not produce swelling, and there is no collapsing of the solidifying cement as in the case of portland cement. On the contrary, the initial strength of the cement becomes higher and the further increase in strength is secured by the development of the process of separation of hydrosilicates and hydroaluminates of calcium, which is stimulated by the formation of sulphoaluminates. It has been established that this run of the process of solidification is related to the concentration of free calcium oxide in the solution and to the presence of calcium sulphate in sufficient quantities.

This theory of solidification and the investigated properties of the sulphate slag cement have found confirmation in practice and are placed by now at the basis of the technology of the manufacturing process.

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GEOLOGY

DO PERMIAN DEPOSITS EXIST ON THE EASTERN SLOPE OF THE URALS?

By N. V. DOROFEEV and D. L. STEPANOV

(Communicated by P. I. Stepanov, Member of the Academy, 16.IV.1944)

Until quite recently the section of the sedimentary Palaeozoic of the Urals has generally been believed to contain no older sediments than the Middle Carboniferous, neither Upper Carboniferous nor Permian deposits having been reported for this section. Within the last years some evidence has been produced, however, pointing to the existence of Permian sediments at several points of the eastern slope of the Urals.

In 1931 N. F. Mamaev discovered a flora of conifers: *Ullmannia frumentaria* Goepp. and *Walchia* sp. nov. in the Kamensk district of the Sverdlovsk region, on the Bagaryak River at the Chaikino village, among marlaceous shales of the variegated series previously referred to the Middle Carboniferous. In 1935, while making new collections, he found there *Gomphostrobus* sp. nov. and *Samaropsis* sp. The above-named forms determined by M. D. Zalessky go to characterize, according to this author, the upper part of the Artinskian or the lower part of the Kungurian stage; on this account Mamaev has referred the Bagaryak variegated series to the Lower Permian (3) . At the same time with Mamaev's first finding, O. F. Neiman, working in the Alapaevsk region, in 1931, referred to the Artinskian stage the areno-argillo-conglomerate formation which had until that time been considered as Middle Carboniferous in age. In 1936 K. N. Dvortzova, from a finding of flora (*Walchia*) assigned to the Lower Permian a thick series of conglomerates and red rocks in the area of the upper course of the Tobol River.

Permian deposits have thus been shown to exist at several points of the eastern slope of the Middle and South Urals, which has duly been reflected on the geological 1 : 500 000 scale map of the Urals and on the corresponding sheets of the geological 1 : 1 000 000 and 1 : 2 500 000 scale maps of the USSR. The discovery of exposures of Lower Permian deposits over such a vast territory made one think that sediments of this age may be extensively developed on the eastern slope of the Urals and in the adjacent part of the West-Siberian lowlands. Hence the palaeogeographical hypothesis and geological-economical prognoses accounting in particular the possible presence of salt and oil in the Permian deposits of the Trans-Ural region *.

In the summer of 1942 the present authors, in the course of geological work, carried on to estimate the possibility of oil occurrences on the eastern slope of the Urals, had an opportunity to visit the above-listed points of exposure of deposits referred to the Permian and some other, and to collect new data on their age. Of greatest interest is the material obtained by one of the authors (N. V. Dorofeev) while carrying on a detailed investigation of the section of the Upper Palaeozoic on the Bagaryak River, in the area located directly west of the Chaikino village.

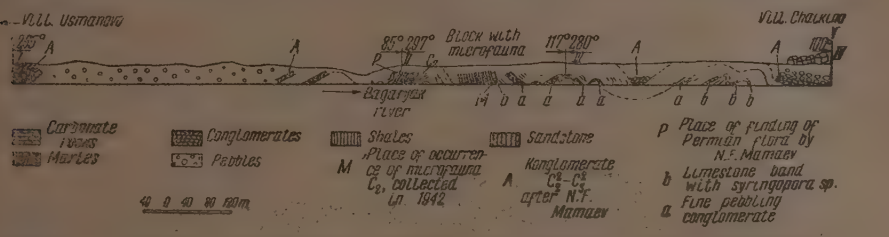
[1] See, *e. g.*, the paper of V. G. Vasiliev (1).

These data showed the tectonics and stratigraphy to be essentially different from those traced by Mamaev (3). In the first place it has been established that the dip in conglomerates C_2^2—C_3(?) (after Mamaev) at the eastern outskirts of the Chaikino village is not a western, as indicated by this writer, but an eastern one: AS. SE 110° at 5—10°. The dip is traceable by the arrangement of pebbles intercalations.

In the west, about 1.5 km from the western outskirts of the Chaikino village, in the C_2^2—C_3(?) conglomerates, according to the description by Mamaev, at the very place where a «NO 50° at 45°» dip is indicated (3), a western dip—AS. NW 290° at 45° has been recorded.

The data obtained were fully in conflict with Mamaev's interpretation of the stratigraphical sequence of the series. It has been found that instead of underlying the variegated series with a Permian flora the C_2^2—C_3(?) conglomerates overly it.

This fact called for a more thorough examination of the section, in order to secure faunistic characteristic of the deposits determined as Lower Permian from their flora. Attention was attracted by a limestone bed with *Chaetetes* sp. and



Section of Bagaryak River between Usmanovo and Chaikino villages (composed by N. V. Dorofeev, 1942)

Syringopora sp. traceable in the upper portion of the section of the variegated series which had also been mentioned by Mamaev. This bed is distinctly seen in the eastern limb of the anticline fold of the variegated series. It is absent in the western limb; neither are the red sandstones and fine-pebbly conglomerates overlying the limestone bed visible there. In exposure 133 of Mamaev on the Bagaryak River we probably have to do with the eastern limb of the fold only, complicated by small folding; the strata with flora seem to belong to deeper horizons than the limestone bands with *Chaetetes* and *Syringopora*. This is in good agreement with the description of the section as given by Mamaev, who does not record their presence west of the beds with flora. From the limestone bed described, crammed with colonies of *Tabulata*, samples were collected, subsequently subjected to a study of microfauna. According to L. P. Grozdilova who has determined the microfauna, the following forms are encountered in the limestone: *Bradyina nautiliformis* Moell., *Glomospira ammodiscoidea* Raus., *Profusulinella* sp. (closely related to *P. parva* Lee et Chen), *P. ovata* Raus., *Schubertella* ex gr. *obscura* Lee et Chen, an abundance of *Globivalvulina* sp., *Archaediscus* sp., *etc.* This complex determines the age of the deposits as the lower part of the Middle Carboniferous, corresponding to the Vereyan or Kashirian horizon of the Moscow basin. In full agreement with this is also the nature of the *Tabulata* fauna, not at all characteristic of the Permian deposits of the Urals.

Thus, the age of the variegated series of the Bagaryak section is determined as the Middle Carboniferous.

The question presents itself: how to bring this conclusion in keeping with the presence in the red beds of a flora of the Rotliegende («Krasny lezhen») type? The present authors adhere to age evaluation based on the study of marine fauna, rather than on palaeophytological evidence; they based their reasoning upon the well-known fact of an intimate relation between the development of land vegetation and the variation in the physical-geographical environment. It has been pointed out by a number of investigators that flora alternations do not infrequently

characterize changes of the landscape rather than age differences. Cases of the presence of flora of a Permian habit in Carboniferous deposits characterized by marine fauna have more than once been reported.

Some time ago beds with marine fauna pointing to their Upper Carboniferous (Pennsylvanian) age have been found in the USA in a formation containing a flora of Permian type (*Walchia*, *etc.*) (5). In the Donetz basin flora of the Rotliegende («Krasny lezhen») is present in the Araucarite series; its reference to the lower part of the Upper Carboniferous may at present be considered as safely established from a study of the marine fauna. Thus, the presence of flora of the Rotliegende («Krasny lezhen») type in itself cannot be considered as discarding the Carboniferous age of the deposits described. Still it should be recognized that the occurrence of this flora in the lower part of the Middle Carboniferous is rather unexpected, since in the cases cited we have to do with its presence in the lower portion of the Upper Carboniferous. All the aforesaid seems to support the views of A. N. Krishtofovich (2) who believes the peculiarities of some Upper Palaeozoic floras to reflect botanical-geographical zoning rather than age differences; the same author believes many continental deposits of Siberia that are usually been considered as Permian, to be synchronous with the Middle Carboniferous of Europe. He finds it «very probable that the Permian flora of Europe (and America) was formed not at the expense of the forms of Westphalian origin, but at the expense of plants which had migrated from the north and north-east, and possibly the north-west, from the Atlantic dry land, where forests of *Walchia* and *Ullmannia* might previously have existed» (2).

Data which make us question the correctness of the age evaluation have been obtained by us also for other regions of the eastern slope of the Urals in which exposures of Permian deposits have been recorded. In the Alapaevsk region N. V. Dorofeev collected samples with microfauna from beds exposed on the left bank of the Alapaevsk pond and assigned by Neiman in 1931 to the Artinskian stage.

The following forms have been determined from these samples by Grozdilova: *Parastaffella* cf. *struvei* (Moell.), *P.* sp. nov., *P.* sp., *Bradyina cribrostomata* Raus. et Reitl., *Archaediscus* cf. *baschkiricus* Krest. et Teod., *A.* sp., *Glomospira* sp., *Textulariidae*, *etc.*, characteristic of the Bashkirian beds and the upper part of the Namurian stage. Thus, the original view of L. S. Librovich who correlated these deposits with the Middle Carboniferous age has been confirmed. It is interesting to note that for the exposures along the Rezh River A. A. Pronin has established, from faunistic data, the Visean age of analogous formations referred by Neiman to the Lower Permian (4).

D. L. Stepanov visited the exposures of deposits referred to the Permian on the eastern slope of the South Urals. The most interesting section of these deposits was encountered on the Tobol River in the region of the Pridorozhny village. Here a thick formation of red polymict conglomerates is exposed. Along with pebbles of igneous rocks, sandstones, siliceous and clayey shales, pebbles and boulders of limestone, frequently containing fauna, are present in abundance. Dvortzova reports the occurrence here of pebbles with Upper Devonian and Tournaisian-Visean fauna. In our material Grozdilova determined in a number of pebbles only Visean microfauna. The conglomerates are overlain by red and violet calcareous schistose sandstones and argillites. In this series Dvortzova discovered a flora including *Walchia*. This finding, along with the lithological habit resembling the Permian red beds of the Western near-Ural region, as well as the resemblance with the variegated series on the Bagaryak River, made her refer the deposits described to the Lower Permian. However, considering the data obtained for the deposits containing a flora including *Walchia* on the Bagaryak River, it seems more probable to assume a Middle Carboniferous age of the red beds on the Tobol River.

This is also supported by the composition of the conglomerate pebbles, among which there are no fragments of rocks younger than the Visean. In the case of a Permian age of the conglomerates it would be more natural to find in them

pebbles of Middle Carboniferous rocks. It may be recalled that the genus *Walchia* can by no means be considered as occurring exclusively in Permian deposits, for it has also been reported in the Stephanian floras of Western Europe.

Analogous data have been obtained also for the exposures of the areno-shale-conglomerate formation on the Karagaily-Ayat denoted on the 1 : 500 000 scale map of the Urals as P_1^A?. In the limestone pebbles of conglomerates of this series we have discovered a Devonian and Lower Carboniferous fauna. No rocks of an age younger than the Visean are present among those pebbles. These rocks can therefore safely be referred to the Middle Carboniferous.

Our observations of 1942 have thus shown that all those deposits of the Urals which are reported in literature as belonging to the Permian are, as a matter of fact, much older, dating apparently from the Middle Carboniferous. In this connexion the question naturally presents itself whether Permian deposits exist on the eastern slope of the Urals at all. We are inclined to answer this question in the negative.

The facial character of the Upper Carboniferous and Permian deposits of the western slope of the Urals shows that the eastern slope and the adjacent part of the West-Siberian lowland had been converted as early as at the end of the Middle Carboniferous from an area of accumulation of sediments to an area of decomposition and removal, supplying with clastic material the territory of the present near-Ural region.

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ECOLOGY

A COMPARISON BETWEEN THE OSMOREGULATORY ABILITY IN VOLGA RIVER AND CASPIAN AMPHIPODS

By G. M. BELÁYEV and J. À. BIRSTEIN

(Communicated by I. I. Schmalhausen, Member of the Academy, 10.III.1944)

The process of penetration of sea organisms into fresh waters has usually been correlated with the osmoregulatory ability of those organisms ([1], [7]). Observations made on the osmotic regulation of animals of marine origin, but inhabiting under normal conditions both sea and fresh-water basins, assume particular importance if considered at this angle. The Caspian basin is known to abound in such species, the most numerous among them being the amphipods which are widely distributed along the rivers of the Ponto-Caspian basin.

Data bearing on the osmotic regulation of Caspian amphipods living under conditions of normal salinity of the Caspian sea (13 pro mille) have already been published by us elsewhere ([3]). At the present time we are in position to compare those data with the results of analogous investigations carried out in June 1940 in the delta of the Volga River. This comparison will make it possible to reveal some peculiar traits of the osmoregulatory process distinguishing species of marine origin when inhabiting fresh-water-basins, from populations inhabiting the sea.

Until recently this problem has scarcely been discussed in literature. On the basis of certain indirect considerations, Medvedeva ([5]) has been led to suppose that in marine *Corophium curvispinum* the internal osmotic pressure is higher than in those inhabiting the Volga River. Pora and Carausu ([6]) have found a lower concentration of NaCl in fresh-water *Pontogammarus maeoticus* as compared to the sea representatives of the same species. Like Medvedeva, the two writers have assumed without any sufficient foundations that fresh-water *Pontogammarus* has a reduced internal osmotic pressure. It is a matter for regret that the assumption made by Medvedeva, Pora and Carausu has not been checked up experimentally; as to our personal experiments, they have failed to corroborate this assumption.

It has been shown by Beadle and Cragg ([2]), moreover, that the distinctions between *Gammarus duebeni* populations dwelling in brackish or fresh-water basins consist in the ability of the latter to maintain in their blood a definite concentration of chlorides when transferred to distilled water; the representatives of the same species dwelling in brackish water are in this case subject to a rapid drop in chloride concentration, which is fatal for them.

In order to find out whether the internal osmotic pressure of the Volga amphipods differs from that of the sea-dwellers belonging to the same species, the internal osmotic pressure has been measured microcryoscopically in eleven species of Volga amphipods. A comparison of the data obtained with those available for sea amphipods ([3]) is to be found in the table below *.

* The data given for each separate species are the means derived from observations carried out on several specimens.

Volga-delta (Δ° of external medium, 0.04°)			Caspian (Δ° of external medium, 0.72°)	
	Δ° of body fluid			
Species	range of individual fluctuations	mean for species	Species	Δ° of body fluid
Dikerogammarus haemobaphes	0.85—0.87	0.86	*D. haemobaphes*	0.91
» *caspius*	1.00—1.05	1.03		
Pontogammarus robustoides	0.93—1.00	0.96	*P. robustoides*	0.87
» *obesus*	0.70—0.95	0.81	*P. obesus*	0.90
» *sarsi*	0.90—1.00	0.95	*P. maeoticus*	0.86
Chaetogammarus tschnus		0.80		
Pandorites platycheir		0.82		
Amathillina cristata	0.93—0.98	0.95		
Gmelina costata	0.78—0.83	0.80		
Corophium chelicorne	0.88—0.97	0.92	*C. chelicorne*	0.88
» *curvispinum*		0.83		
Mean	0.70—1.05	0.88	Mean	0.88

It may be seen from this table that the internal osmotic pressure in sea amphipods inhabiting the lower reaches of the river has not been affected by their transfer from sea to fresh water. The deviations recorded for the representatives of a single species taken from the Volga or from the sea do scarcely exceed the range of individual variation. Thus, the individual variations recorded in the internal osmotic pressure for *Pontogammarus obesus* range within 0.70—0.95°. The stability of the internal osmotic pressure and its independence of the osmotic pressure of the external medium point to the existence of a definite optimum of salinity of the body fluid in the Volga-Caspian amphipods, which is necessary for their normal life activity.

No less interesting seemed to us to investigate the peculiar traits of the osmoregulatory process in fresh-water populations under conditions of varying salinity of the external medium, as compared to those of sea populations. Similarly as in our previous studies (3), the experimental animals were kept during three days in Caspian water of various salt concentrations, after which their internal osmotic pressure was subjected to measurements. Unfortunately, we were able to compare data on sea- and fresh-water amphipods thus obtained within a single species only, *viz.* in *Dikerogammarus haemobaphes* (Fig. 1). From a comparison of the curves plotted the following conclusions have to be derived:

1. The general run of the curves of the variation of the internal osmotic pressure in the case of varying salinity of the external medium is essentially the same, both in fresh and sea-water *Dikerogammarus*; the part of the curve corresponding to rather low concentrations of salinity of the external medium shows in either case a rather gentle slope with respect to the axis of the abscissa, while in the part corresponding to higher salt concentrations the curve makes a kink, after which it runs parallel to the curve plotted for the osmotic pressure of sea water.

2. The left-hand gentle slope of the curve plotted for fresh-water *Dikerogammarus* shows a similar angle of inclination, it runs, however, somewhat above the curve for the sea-dwellers. This should have been anticipated, since the normal internal osmotic pressure recorded in the former case in fresh water is Δ°—0.04°, while in the latter case, at a salinity of 13 pro mille, the respective figures are Δ°—0.72°; the angle of inflection in the left-hand part of the curve remains constant and seems to be characteristic for the given species.

3. With sea-*Dikerogammarus* the internal osmotic pressure measured at a salinity below 5 pro mille (Δ°—0.26°) is subject to sharp variations (3); nothing else

than an hypothetical curve can be plotted for such low salt concentrations. We have been able, however, to record a rather constant internal pressure in the Volga-representatives of the species when kept in fresh water, which is responsible for the curve running at a constant angle of inflection, from 0 to the point of the kink.

The peculiarity described above has been found to be inherent also in two other Volga-amphipods, investigated by us for the relation between their internal osmotic pressure and that of the external medium, under conditions of varying salinity of the external medium (*Pontogammarus robustoides* and *Corophium chelicorne*) (Fig. 2). We feel inclined to believe this type of curve to be typical of the euryhaline Volga amphipods, in general. The stenohaline Caspian amphipods (*P. maeoticus*) display a sharply reduced internal osmotic pressure at about 5 pro

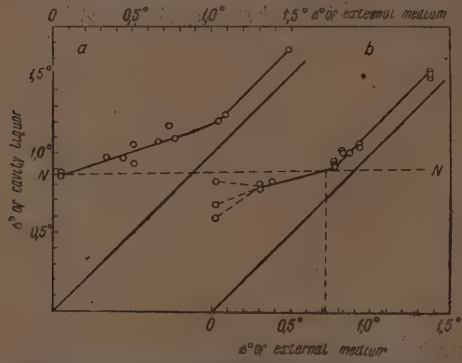

Fig. 1. Correlation between internal and external osmotic pressure in Volgian (*a*) and Caspian (*b*) *Dikerogammarus haemobaphes*, in Caspian water of varying salinity, after 3 days. *N*—*N*, Δ° of body fluid of the normal medium

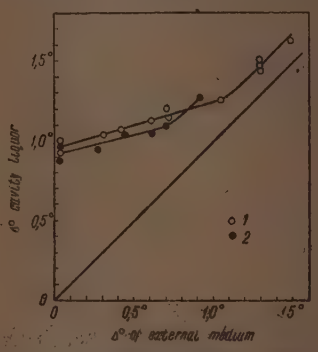

Fig. 2. Correlation between internal and external osmotic pressure in Volgian *Pontogammarus robustoides* (*1*) and *Corophium chelicorne* (*2*) in Caspian water of varying salinity, after 3 days

mille salinity, which is responsible for a kink in the curve of the relation between the internal and external osmotic pressure at the point corresponding to this concentration(8).

If all that has been said here proves to hold true, it should be assumed that a certain part of populations of sea amphipods, within which the highest indices of internal osmotic pressure after transfer to fresh water have been recorded, are physiologically adapted for invasion into fresh water. This state of the affair seems to be in agreement with the opinion advanced by the adherents of the theory of preadaptation, which implies the appearance in sea animals of adjustments to the dwelling in fresh water prior to their migration to fresh water (7). We feel, however, rather inclined to put down the remarkable euryhalinity of many a Caspian amphipod to the influence of the freshening of the sea, caused by an afflux of thawing waters of the receding glacier.

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PLANT PHYSIOLOGY

ON THE SYNTHESIS OF RUBBER BY PLANTS

III. PHOTOSYNTHESIS AND RUBBER FORMATION IN PLANTS

By A. A. PROKOFIEV

(Communicated by A. N. Bach, Member of the Academy, 18.IV.1944)

As has been shown in our previous work (1), the accumulation of rubber in the laticiferous vessels of plants can proceed in the absence of light. By the method of sterile cultures (2) it is possible to raise tau-saghyz on carbohydrates during several months in the dark. Notwithstanding the absence of light, the laticiferous system developing in the plant's organs becomes filled with latex containing multitudes of rubber globules. The fact of rubber formation in the dark is by no means unique: it is well known, for example, that during the germination of seeds of root-rubber plants the laticiferous vessels containing rubber are formed before the plant passes to photosynthetic activity. The exudation of considerable quantities of rubber in the case of tapping of tau-saghyz plants deprived of their above-ground parts also points to that the generation of rubber is not directly connected with photosynthesis. The formation of rubber in the ethiolated leaves of guajula kept in the dark (3) goes to confirm the same conclusion. It is thus evident that the production of rubber does not cease even in a total absence of photosynthesis. Our earlier assumption as to the rubber synthesis being a reaction requiring no radiant energy should be considered as definitely proved (1). Nevertheless, denying the existence of a direct connexion between the photosynthetic activity of the assimilating apparatus and the formation of rubber in the roots, we have to admit, however, that the products of photosynthesis serve as a *matière première* for rubber formation. Consequently, we have to decide whether any specific precursors of rubber appear in the rubber plant during the process of photosynthesis (4, 5) or the specific traits of the rubber plant lie outside the photosynthesis. In the latter case rubber plants and plants producing no rubber would be identical in principle with respect to the metabolism taking place in their leaves. The peculiar features of metabolism, leading in some plants to rubber formation, would then consist in the specific mode of transformation of common assimilates into vital products. As root-rubber plants are forming the main part of their rubber in the roots, it is in this organ that the said peculiarities of metabolism of these plants are to be looked for.

In order to throw some light upon these problems I have started special grafting experiments (1). By comparing the results obtained with various combinations of rubber plants with rubberless plants one could expect to succeed in evaluating the part played by the different organs of the plant in the production and accumulation of rubber. In fact, if specific precursors of rubber arise in the process of photosynthesis in rubber plants, the rubber content of a non-rubber dandelion would be likely to increase considerably in the case kok-saghyz be grafted to it. On the contrary, if the dandelion were used as a scion, and kok-saghyz serves as

a stock, we had to expect a sharp drop in the rubber content of the root of the stock, *i. e.* of kok-saghyz.

Such combinations between rubber-bearing plants and dandelions containing scarcely any rubber were realized by us in 1941 and 1942. A total of over 600 grafts have been obtained in the following combinations:

Scion / stock								
1 Kok-saghyz / Dandelion	2 Dandelion / Kok-saghyz	3 Kok-saghyz / Kok-saghyz
4 Dandelion / Dandelion	5 Dandelion / Krym-saghyz	6 Krym-saghyz / Dandelion
7 Krym-saghyz / Krym-saghyz	8 Krym-saghyz / Kok-saghyz	9 Kok-saghyz / Krym-saghyz

Combinations 3, 4 and 7 served as controls, showing the accumulation of rubber in the roots supplied with the assimilates from the same plant species.

In our experiments of 1941 we made use of seedlings raised in a greenhouse early in spring; many of the graft combinations flowered during the same season and produced good seed. In the experiments of 1942 graftings were made as late as in the second half of July, so that towards the winter they have developed only vegetative organs. In this latter case, however, the graftings had similarly pretty well developed above- and underground parts. The scions have formed towards the autumn rather robust rosettes, consisting of many dozens of leaves. The stocks developed numerous roots, many of which were 4—5 mm in diameter. These newly formed roots of the grafts were subjected to analysis. It is of interest to note that the nature of the scion did not influence the morphology of the roots of the stock. Thus, for example, the common dandelion developed, irrespective of the nature of the scion (kok-saghyz, krym-saghyz or dandelion), a highly ramified root system, generally typical of this plant-species. And, conversely, in the case of krym-saghyz which has usually an axial root, this type of the root system has been preserved in the graftings, whatever the kind of scion. An anatomical examination of the newly arising roots of the stocks showed that in the control combinations Nos. 3, 4 and 7 neither the structure of the roots nor the number of laticiferous vessels pervading them differed appreciably from that of the roots belonging to ungrafted plants of the same species. A more interesting picture is offered by heterogeneous grafts. Thus, for example, microscopic examination of the union of the components of the graft kok-saghyz—dandelion shows that the united laticiferous system arising after the growing together of stock and scion is of different thickness in different parts of the graft: in the kok-saghyz component the laticiferous vessels are large and well filled with rubber, while in the dandelion component they are small and poor in rubber. An analysis of the roots of the stock showed that the rubber content inherent in the plant in question is maintained irrespective of the kind of the scion. Thus, *e. g.*, the roots of a kok-saghyz plant that had received products of photosynthesis from the leaves of dandelion, accumulated rubber quite normally. The dandelion roots which had been supplied from leaves of kok-saghyz, remained in spite of this poor in rubber. Similar results were obtained with the combinations 5 and 6.

The evidence reported here makes us question the correctness of Nichiporovich's hypothesis (4,5) as to the synthesis of specific rubber precursors in leaves. The opposite assumption is by far more probable, *viz.* that no such precursors exist in the leaves of rubber plants. Hence it should be concluded that any aboveground part must ensure the formation of rubber in the roots of the stock belonging to rubber-yielding species.

In order to check up this supposition, root-cuttings of *Lappa* were grafted to root-cuttings of kok-saghyz and krym-saghyz*. As in the other combinations, young

* Complete lack of rubber in the various species of *Lappa* has been proved during the revision of the USSR vegetation for rubber plants.

plants raised from seed for this special purpose were used for graftings. It should be pointed out that in the case of the combination *Lappa*—krym-saghyz, the growing together of the components and the further growth proved much inferior to those in the case of graftings within the genus *Taraxacum*. Nevertheless, about 10 per cent of the grafted plants survived until the autumn dugging. External examination of these grafts showed that the grafted parts had grown together. The parts of the stock (kok-saghyz, krym-saghyz) were found to have developed abundant young roots which did not exceed in thickness one millimeter, however. Such a weak root system of the rubber plants seemed to be unable fully to supply the aboveground organs of *Lappa* (3—4 leaves), so that the latter had to produce several roots of its own. Anatomical examination went to confirm the fact of the growing together of the tissues of the scions, a more perfect union taking place in the combination *Lappa*—krym-saghyz. The young roots of the rubber-yielding stocks which had developed at the expense of the assimilates of *Lappa*, were analysed for their rubber content. It has been found in the first place, that the latex flows out at the points of the injury. The rubber globules contained in the latex were in shape and size typical of young roots of the corresponding plant-species. Microchemical analysis of the young roots of the rubber-yielding stocks (kok-saghyz and krym-saghyz) has similarly proved beyond any doubt the presence of rubber in the laticiferous vessels*.

Inasmuch as *Lappa* is wholly deprived of rubber, there is no ground to suppose that its leaves would form any sort of rubber precursor. Nevertheless, we are facing here an irrefutable instance of rubber formation in the root of kok-saghyz, taking place at the expense of the photosynthetic products which were supplied to the roots of these species from the leaves of *Lappa*.

The results reported in this note are evidence of the fact that rubber formation in the roots of a plant does not depend on the nature of the assimilating organs. And this should be interpreted to mean that no specific rubber precursors are ever formed during the process of photosynthesis in the leaves of rubber-plants.

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* The presence of rubber was detected by the bromination method [6].

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HELMINTHOLOGY

A CONTRIBUTION TO THE STUDY OF THE DEVELOPMENT CYCLE IN THE CESTODE *MESOCESTOIDES LINEATUS* (GOEZE, 1782), PARASITIC OF CARNIVOROUS MAMMALS

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Notwithstanding the numerous attempts that have been made to trace the cycle of development of *Mesocestoides lineatus* (Goeze, 1782), a tape-worm parasitic of *Carnivora*, the problem has as yet not been definitely solved.

Leuckart (1) became aware of the resemblance between the head of *Tetratyridiae* from the abdominal cavity of a newt and the scolex of *Mesocestoides litteratus* (Batsch, 1786) from the intestine of a fox. Subsequent investigations (2–7) have proved that *Tetratyridiae* found in various vertebrates are nothing else than the larval stage of *Mesocestoides lineatus*.

Henry (5) and Witenberg (7), however, arrived at a conclusion that *Tetratyridiae* are not the unique larval stage of *Mesocestoides*.

Thus, the developmental cycle of this cestode cannot proceed unless two hosts participitate in it, an intermediate and a supplementary one; out of these, the intermediate host has still remained unknown, while the supplementary ones are just those various species of mammals, birds and reptiles in whose body cavities larvae of *Mesocestoides* at the stage of *Tetratyridiae* have often been found to be localized.

With the purpose to find out the intermediate host of *M. lineatus*, Henry(5) and Witenberg (7) were feeding eggs of *Mesocestoides* to flies (*Musca domestica*, *Calliphora* sp.) and to blackbeetles, without having obtained any positive results, however. Joyeux and Koboziev (cited after Witenberg (7)) found in beetles *Geotrupes sylvaticus* cysticercoids whose morphological characters reminded those of *Mesocestoides*; this correlation has not, however, been proved experimentally.

In the work here reported I have made an attempt to detect the primary larval stage of *M. lineatus* in the moss-mites of the group *Oribatei*. I have been prompted to this investigation by the recent literary data, according to which moss-mites of the group of *Oribatei* often serve as the intermediate hosts for various species of tape-worms of the order *Anoplocephalata*.

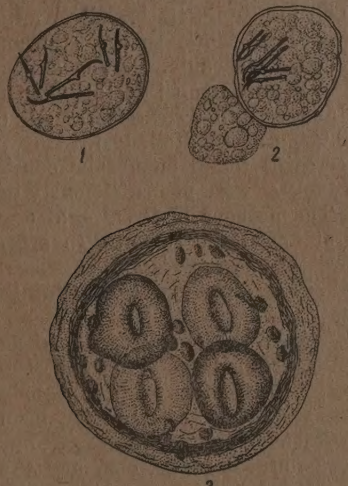
The work was carried on along two lines: a) searching for oribatids naturally invaded by cysticercoids of *M. lineatus*, and b) experimentally infesting moss-mites with eggs of *M. lineatus*.

1. Search for Larvae of *M. lineatus* in Mites Naturally Infested. As material for diagnosticating natural invasion use was made of mites collected on the territory of a fox-breeding farm; the silver-black foxes of this farm were badly invaded by helminths and periodically eliminated large numbers of mature segments of *M. lineatus*. To analysis was subjected substratum taken from between the cages, within the cages, scraped from the earthern floor and from under the wooden floor of some of the cages. The mites were extracted of the substratum with the aid of Tullgren's apparatus.

A total of 775 specimens of *Oribatei* mites were collected and subjected to dissection. As a result of these dissections, 12 mites have been found to contain cestodes at various developmental stages.

In some of the larval stages I found hooks typical of *M. lineatus* (the middle pair longer than the rest). This fact, along with the absence at the fox-farm of any domestic animals, made me suppose that the larvae discovered in the body cavities of the *Oribatei* should be referred to *M. lineatus*.

In mites referring to the genus *Trichoribates* two oncospheres have been found, oval in shape, 0.025—0.036 mm in diameter, armed with six mobile hooks typical of *M. lineatus* (see figure, *1*). Moreover, in mites of the same genus I found two larvae 0.025—0.035 mm across, with immobile hooks and with a clearly detectable protrusion of cells at one of the poles, the initiation of cercomeres (figure, *2*).



1, Oncosphere from *Trichoribates* sp. *2*, Oncosphere from *Trichoribates* sp., initial stage of cercomere formation. *3*, Cysticercoid from *Scheloribates latipes*. Larval stages discovered in mites experimentally infested

I also happened to find four larvae spherical in shape, their maximum body size reached 0.040 mm in diameter; the maximum length of a fully developed caudal appendix reached 0.024 mm, against a breadth of 0.016 mm. Aside from these findings, I discovered in *Trichoribates* sp. larvae at the stage of cysticercoid, 0.104 mm across, without a caudal appendix, with suckers 0.025 mm across. In body cavities of *Scheloribates laevigatus* and *Punctoribates* sp. elongated larvae have also been found; their maximum body length attained 0.088—0.128 mm, against a breadth of 0.060 mm in the narrowest part of their body, and 0.024—0.028, in the broadest. At the anterior end accumulations of cells could clearly be detected, with traceable outlines of the future suckers. No hooks could be detected. The stage of cysticercoid has also been found in the body cavity of *Sch. latipes* (figure, *3*). The cysticercoid was spherical in shape, reaching 0.10 mm in diameter. The solid external membrane of its body was 0.004 mm thick, while the internal, fibrous one had a thickness of 0.002 mm. Within the cysticercoid four powerful suckers could clearly be seen, 0.022 mm in diameter.

2. Formation of *M. lineatus* Larvae in Mites Experimentally Infested. We practised experimental infestation of *Oribatei* ticks with eggs of *M. lineatus*.

A total of 3000 specimens of mites have been experimentally invaded and afterwards dissected; they belonged to the following four families: a) fam. *Ceratozetidae*, b) fam. *Galumnidae*, c) fam. *Natospididae*, d) fam. *Liacaridae*.

When dissecting *Trichoribates* on the 5th and 10th day after infestation, in the body cavity of four specimens were discovered oncospheres of *M. lineatus*, oval in shape, 0.032 × 0.028 mm across, armed with six clearly detectable and fairly mobile hooks. When dissecting the experimental mites at later periods, larvae of larger dimensions were also encountered, assuming a rather rounded shape. Spherical larvae of *M. lineatus* were discovered in three specimens of *Trichoribates* dissected at the 30th day upon infestation. Those larvae reached 0.040—0.044 mm in diameter; the cercomeres were 0.024—0.028 mm long and 0.016—0.020 mm broad. At a still later developmental stage the larvae had an elongated body shape. In a *Sch. latipes* mite dissected 58 days upon infestation, there was found a larva 0.090 mm long; its broadest, anterior body part reached 0.060 mm, the narrowest was 0.028 wide. In the thickened part of its body the outlines of forming suckers were traceable. Larvae at the stage of cysticercoids were discovered at the 122—125 day upon infestation in the body cavity of *Sch. laevigatus* (0.094 mm in diameter), in *Trichoribates* sp. (0.108 mm in diameter) and in *Punctoribates* sp. (0.090 mm in diameter).

The stages of the cysticercoids found cannot be considered as fully formed, invasional stages, for their suckers were not sufficiently developed and the scolex was not fully delimitated. The absence of invasional cysticercoids prevented us from infesting the additional hosts, the rodents, in order to obtain the second larval stage of *Mesocestoides*, the *Tetratyridium*.

The investigation carried out justifies the conclusion that the first intermediate hosts of *M. lineatus* are small soil *Oribatei* mites.

It is possible that, aside from the *Oribatei*, *M. lineatus* has also some other intermediate hosts. It does not seem improbable that the *Oribatei* are rather the facultative than the obligatory intermediate hosts of this helminth. We are facing, however, an irrefutable fact which has never as yet been reported in literature, *viz.* that *Oribatei* mites are subject to the invasion with oncospheres of *M. lineatus* both under experimental and natural conditions, and that in the body cavity of the *Oribatei* these oncospheres develop into cysticercoids. The *Oribatei* are thus to be ascribed an active rôle in the epizootology of the mesocestoidosis in carnivorous mammals.

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